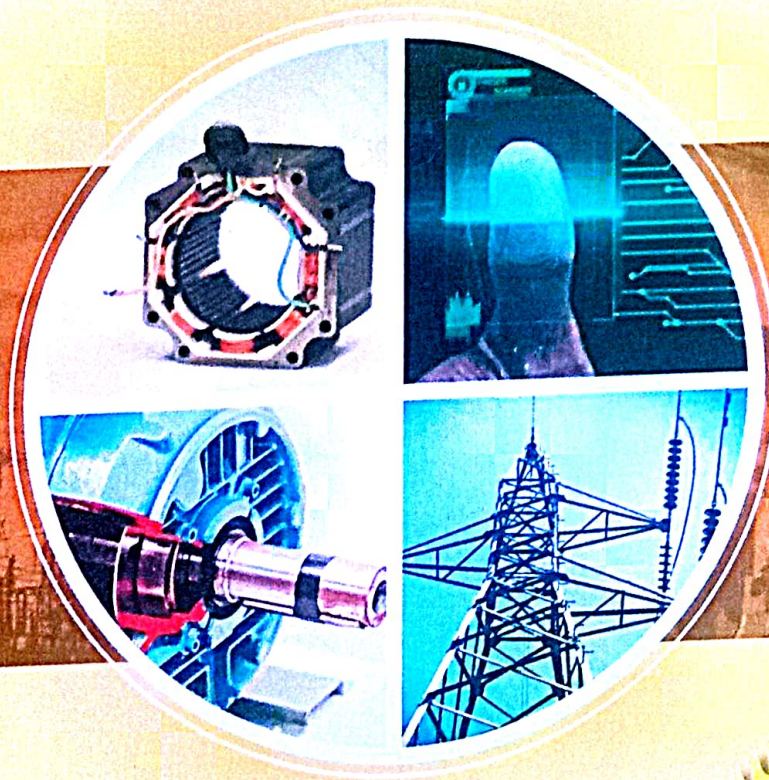


QUANTUM *Series*

Semester - 5

Electrical & Electronics Engineering

Control System



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**Session
2024-25
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सजग सूचना

छात्रों, डीलरों, दुकानदारों, कॉपियर्स और जिनसे यह संबंधित हो सकता है क्वांटम पुस्तकों के कॉपीराइट उल्लंघन के संबंध में दिल्ली के माननीय उच्च न्यायालय का आदेश

यह छात्रों, डीलरों, दुकानदारों, कॉपियर्स और आम जनता को सूचित करना है कि क्वांटम पेज प्राइवेट लिमिटेड पुस्तकों की क्वांटम श्रृंखला में कॉपीराइट का मालिक है।

क्वांटम पुस्तकों (चाहे सॉफ्ट कॉपी या हार्ड कॉपी में) की कोई भी अनधिकृत प्रतिलिपि, स्कैनिंग, पुनरुत्पादन, वितरण या संचलन, क्वांटम पेज प्राइवेट लिमिटेड के कॉपीराइट और ट्रेडमार्क अधिकारों के उल्लंघन के बराबर है, जो दिवानी अपराध के साथ-साथ दंडिक अपराध है, जिनकी सजा कारावास तक है।

क्वांटम पेज प्राइवेट लिमिटेड ने दिल्ली के माननीय उच्च न्यायालय के समक्ष क्वांटम पुस्तकों के उल्लंघनकर्ताओं के खिलाफ क्वांटम पेज प्राइवेट लिमिटेड बनाम टेलीग्राम एफजेड एलएलसी और अन्य, सीएस (कॉम) १२१/२०२२ नामक एक मुकदमा शुरू किया है। 23 दिसंबर, 2022 के अपने आदेश में, माननीय उच्च न्यायालय ने माना है कि क्वांटम पुस्तकों का अनधिकृत पुनरुत्पादन कॉपीराइट उल्लंघन के बराबर है और टेलीग्राम चैनलों सहित विभिन्न स्रोतों से क्वांटम पुस्तकों की उल्लंघनकारी प्रतियों को हटाने का निर्देश दिया है।

छात्रों, डीलरों, दुकानदारों, कॉपीयरों और आम जनता को एतद्वारा आगाह किया जाता है कि वे क्वांटम पुस्तकों (चाहे सॉफ्ट कॉपी या हार्ड कॉपी में) की कोई भी अनधिकृत नकल, स्कैनिंग, पुनरुत्पादन, वितरण और वितरण न करें।

क्वांटम पुस्तकों के ऐसे किसी भी अनधिकृत उपयोग से उल्लंघनकर्ताओं पर क्वांटम पेज प्राइवेट लिमिटेड द्वारा दीवानी और आपराधिक कार्यवाही शुरू की जाएगी।

QUANTUM SERIES

For

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CONTROL SYSTEM

By
Varun Gupta



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1 UNIT

Control System Concepts

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1-2 C (EN-Sem-5)

Control System Concepts

PART-1

Elements of Control Systems.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. What are the basic elements of control system ?

Answer

The various elements of closed loop system are shown in Fig. 1.1.1.

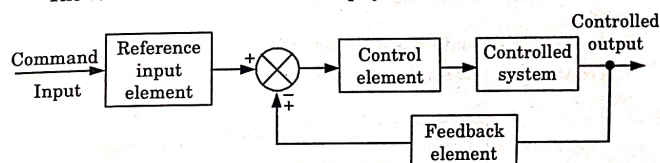


Fig. 1.1.1.

- 1. Command :** The command is the externally produced input and independent of the feedback control system.
- 2. Reference input element :** This produces the standard signals proportional to the command.
- 3. Error detector :** The error detector receives the measured signal and compares it with reference input. The difference of two signals produces the error signal.
- 4. Control element :** This regulates the output according to the signal obtained from error detector.
- 5. Controlled system :** This represents what we are controlling by the feedback loop.
- 6. Feedback element :** This element feedback the output of the error detector for comparison with the reference input.

PART-2

Concept of Open Loop and Closed Loop Systems, Examples and Application of Open Loop and Closed Loop Systems.

1-1 C (EN-Sem-5)

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.2. Explain open loop and closed loop control system with the help of suitable examples.

Answer

A. Open loop control system :

- The open loop control system is also known as control system without feedback or non-feedback control system.
- In open loop systems the control action is independent of the desired output.
- In this system the output is not compared with the reference input.
- The component of the open loop systems are controller and controlled process.

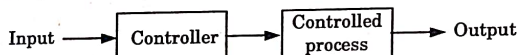


Fig. 1.2.1.

5. Examples :

- Immersion rod
- A field control DC motor
- Door lock system
- Traffic control system.

B. Closed loop control system :

- Closed loop control system is also known as feedback control system.
- In closed loop control system the control action is dependent on the desired output.

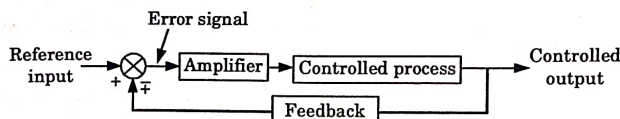


Fig. 1.2.2.

- Any system having one or more feedback paths forms a closed loop system. In closed loop systems the output is compared with the reference input and error signal is produced.

- The error signal is fed to the controller to reduce the error and desired output is obtained.

5. Examples :

- Air conditioners
- Autopilot aeroplane
- Electric iron.

Que 1.3. Compare the open loop control system and closed loop control system.

Answer

S. No.	Open loop	Closed loop
1.	Feedback is not present. So any change in output has no effect on the input.	Feedback is present. So changes in output effects input.
2.	It is not much accurate.	It is very accurate.
3.	It is very sensitive to errors and disturbances.	Less sensitive to errors and disturbances.
4.	It has small bandwidth.	It has large bandwidth.
5.	Simple in construction and is cheap.	Complicated in design and costly.

PART-3

Mathematical Modelling of Physical Systems (Electro-mechanical).

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.4. Derive force-voltage analogy.

Answer

A. Translation mechanical systems :

- A translational spring-mass-damper system is shown in Fig. 1.4.1.

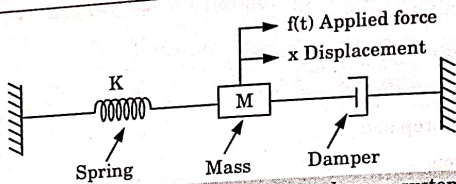


Fig. 1.4.1. Translational spring-mass-damper system.

2. Here,
 M = Mass of system
 $f(t)$ = Applied force
 D = Coefficient of damping
 K = Spring deflection constant
3. The equation of motion for the system is obtained by applying D' Alembert's principle,

$$M \frac{d^2 x}{dt^2} = -D \frac{dx}{dt} - Kx + f(t) \quad \dots(1.4.1)$$

4. Rearranging eq. (1.4.1), we get

$$M \frac{d^2 x}{dt^2} + D \frac{dx}{dt} + Kx = f(t) \quad \dots(1.4.2)$$

B. Force-voltage analogy :

1. The analogy of eq. (1.4.2) with the voltage equation of an electrical circuit can be established by considering electrical circuit shown in Fig. 1.4.2.

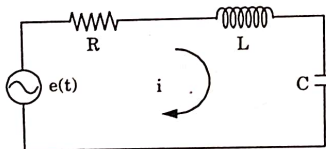


Fig. 1.4.2. Electrical circuit for force-voltage analogy.

2. The voltage equations for the circuit is as follows :

$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt = e(t) \quad \dots(1.4.3)$$

3. As the current is the rate of flow of electric charge,

$$i = \frac{dq}{dt} \quad \dots(1.4.4)$$

4. Substituting eq. (1.4.4) in eq. (1.4.3),

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = e(t) \quad \dots(1.4.5)$$

5. Comparing eq. (1.4.5) and (1.4.2), we get

Table 1.4.1. Force-Voltage Analogy

Mechanical system	Electrical system
Force, f	Voltage, e
Velocity, v	Current, i
Displacement, x	Charge, q
Mass, M	Inductance, L
Damping coefficient, D	Resistance, R
Compliance, $1/K$ (stiffness, K)	Capacitance, C

Que 1.5. Derive force-current analogy.

Answer

- A. Translational mechanical system :** Refer Q. 1.4, Page 1-4C, Unit-1.
 Equation of spring-mass-damper system

$$M \frac{d^2 x}{dt^2} + D \frac{dx}{dt} + Kx = f(t) \quad \dots(1.5.1)$$

B. Force-Current analogy :

1. The nodal equation as obtained for the circuit shown in Fig. 1.5.1 is analogous to spring-mass-damper system described by the eq. (1.5.1).

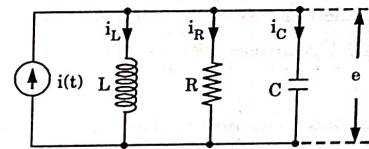


Fig. 1.5.1. Electrical circuit for force-current analogy.

2. According to nodal analysis
 $i_L + i_C + i_R = i(t) \quad \dots(1.5.2)$

3. The common voltage across the three parallel elements of the circuit is denoted as e . The following relations are obtained :

$$i_L = \frac{1}{L} \int e dt \quad \dots(1.5.3)$$

$$i_R = \frac{1}{R} e \quad \dots(1.5.4)$$

$$i_C = C \frac{de}{dt} \quad \dots(1.5.5)$$

4. Substituting eq. (1.5.3), (1.5.4) and (1.5.5) in eq. (1.5.2),

$$\frac{1}{L} \int e dt + \frac{1}{R} e + C \frac{de}{dt} = i(t) \quad \dots(1.5.6)$$

5. The voltage e is related to flux linkages associated with inductance L as follows:

$$e = \frac{d\phi}{dt} \quad \dots(1.5.7)$$

6. Putting $e = \frac{d\phi}{dt}$ in eq. (1.5.6), we get

$$\frac{1}{L} \phi + \frac{1}{R} \frac{d\phi}{dt} + C \frac{d^2\phi}{dt^2} = i(t)$$

$$C \frac{d^2\phi}{dt^2} + \frac{1}{R} \frac{d\phi}{dt} + \frac{1}{L} \phi = i(t) \quad \dots(1.5.8)$$

7. Comparing eq. (1.5.1) and (1.5.8), we get

Table 1.5.1. Force-Current Analogy

Mechanical system	Electrical system
Force, f	Current, i
Velocity, v	Voltage, e
Displacement, x	Flux linkage, ϕ
Mass, M	Capacitance, C
Damping coefficient, D	Conductance, G
Stiffness, K (compliance, $1/K$)	Inductance, L

Que 1.6. Write down the differential equation governing the mechanical translation system and find the transfer function.

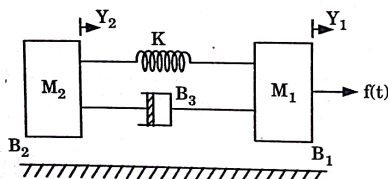


Fig. 1.6.1.

AKTU 2020-21, Marks 10

Answer

1. Free body diagram of M_2 :

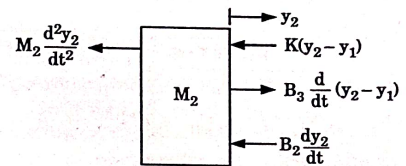


Fig. 1.6.2.

$$M_2 \frac{d^2y_2}{dt^2} + K[y_2 - y_1] + B_3 \frac{d}{dt} [y_2 - y_1] + B_2 \frac{dy_2}{dt} = 0 \quad \dots(1.6.1)$$

2. Free body diagram of M_1 :

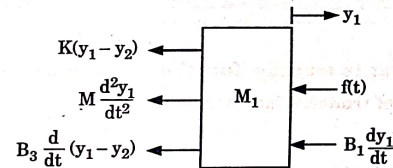


Fig. 1.6.3.

$$f(t) = M_1 \frac{d^2y_1}{dt^2} + B_3 \frac{d}{dt} [y_1 - y_2] + K[y_1 - y_2] + B_1 \frac{dy_1}{dt} \quad \dots(1.6.2)$$

3. Taking laplace transform of both sides of eq. (1.6.2) and eq. (1.6.1), we get

$$F(s) = M_1 s^2 Y_1(s) + B_3 s [Y_1(s) - Y_2(s)] + K[Y_1(s) - Y_2(s)] + B_1 s Y_1(s) \quad \dots(1.6.3)$$

$$M_2 s^2 Y_2(s) + K[Y_2(s) - Y_1(s)] + B_3 s [Y_2(s) - Y_1(s)] + B_2 s Y_2(s) = 0$$

$$(M_2 s^2 + K + B_3 s + B_2 s) Y_2(s) = K Y_1(s) + B_3 s Y_1(s)$$

$$Y_2(s) = \frac{(K + B_3 s) Y_1(s)}{M_2 s^2 + K + B_3 s + B_2 s}$$

4. Putting the value of $Y_2(s)$ in eq. (1.6.3)

$$F(s) = (M_1 s^2 + B_3 s + K + B_1 s) Y_1(s) - (B_3 s + K) Y_2(s)$$

$$F(s) = (M_1 s^2 + B_3 s + K + B_1 s) Y_1(s) - \frac{(B_3 s + K)(B_3 s + K) Y_1(s)}{M_2 s^2 + K + B_3 s + B_2 s}$$

$$\frac{F(s)}{Y_1(s)} = \frac{(M_1 s^2 + B_3 s + K + B_1 s)(M_2 s^2 + K + B_3 s + B_2 s) - (B_3 s + K)^2}{(M_2 s^2 + K + B_3 s + B_2 s)}$$

$$\frac{Y_1(s)}{F(s)} = \frac{M_2 s^2 + K + B_3 s + B_2 s}{(M_1 s^2 + B_3 s + K + B_1 s)(M_2 s^2 + K + B_3 s + B_2 s) - (B_3 s + K)^2}$$

PART-4

Determination of Transfer Function by Block Diagram Reduction Techniques and Signal Flow Method using Mason's Gain Formula, Basic Characteristics of Negative Feedback Control Systems.

Questions-Answers**Long Answer Type and Medium Answer Type Questions**

Que 1.7. What is transfer function? Write advantages and disadvantages of transfer function.

Answer

A. Transfer function : Transfer function is defined as the Laplace transform of output to the Laplace transform of input when all initial conditions are set to zero.

$$T(s) = \frac{L[c(t)]}{L[r(t)]} = \frac{C(s)}{R(s)} \quad \text{All initial condition are zero}$$

B. Advantages :

1. Stability analysis of the system can easily be carried out.
2. The use of Laplace transform approach allows converting integral-differential time domain equations to simple algebraic equations.

C. Disadvantages :

1. It is applied only to linear and time invariant systems.
2. Initial conditions do affect the system performance but these are neglected while determining transfer function and hence lose their importance.

Que 1.8. Define blocks, summing point and branch point. Explain the block diagram reduction rules.

OR

Write down various rules involve in block diagram reduction method.

AKTU 2021-22, Marks 10

Answer**A. Blocks :**

1. A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals.
2. A system is represented by using block diagrams. These are easy to construct even for the complicated system. It is easy to visualize the function of individual element in the block diagram of a system.

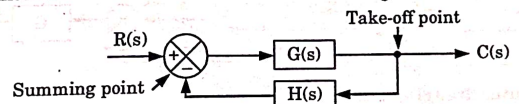
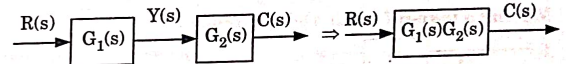
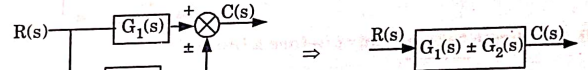
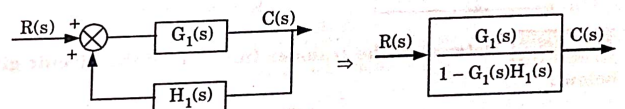
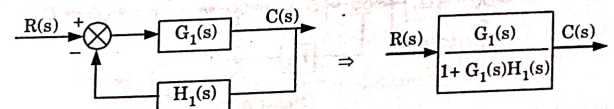


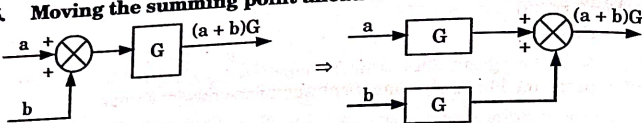
Fig. 1.8.1. Schematic diagram of closed loop system.

B. Summing point : The output of control system is feedback to the summing point where it is compared with the reference input.

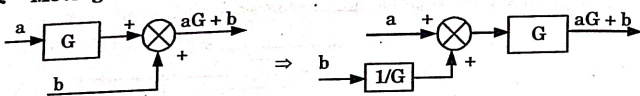
C. Branch point : A branch point or a take off point is a point from which the signal from a block goes concurrently to other blocks or running points.

D. Block diagram reduction rules :**1. Series connection :****2. Parallel connection :****3. Positive feedback connection :****4. Negative feedback connection :**

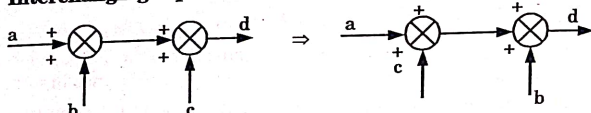
5. Moving the summing point ahead of the block :



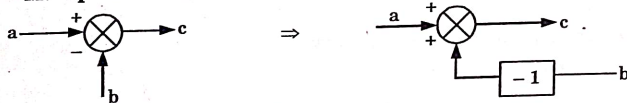
6. Moving the summing point before the block :



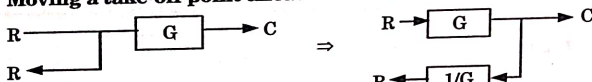
7. Interchanging input :



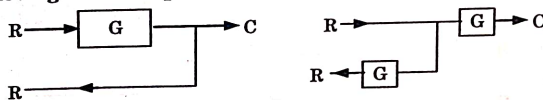
8. Multiplication by (-1) to the input :



9. Moving a take-off point ahead a block :



10. Moving a take-off point before a block :



Que 1.9. Determine the transfer function of the circuit given below :

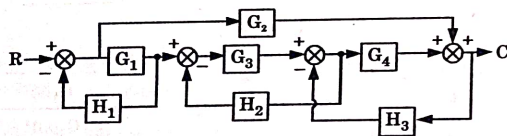


Fig. 1.9.1.

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Answer

Step 1 :

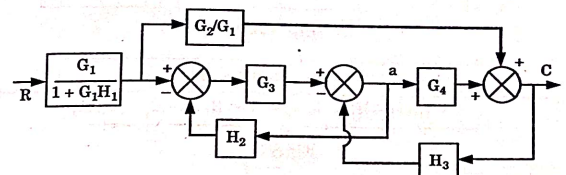


Fig. 1.9.2.

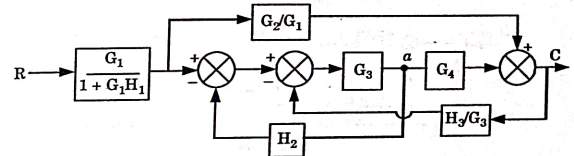
Step 2 : Shift summing point a before G_3 .

Fig. 1.9.3.

Step 3 : Interchange summing points.

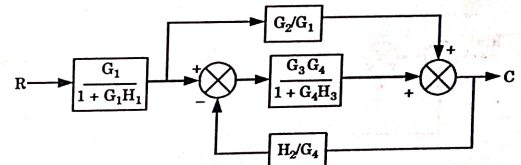


Fig. 1.9.4.

Step 4 : Shift summing point before block.

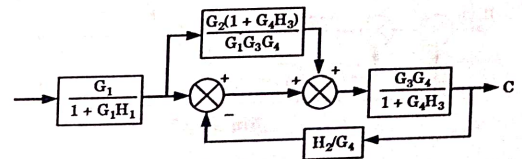


Fig. 1.9.5.

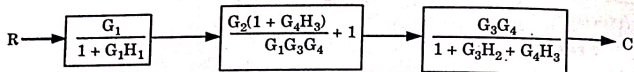
Step 5 : Interchange summing points.

Fig. 1.9.6.

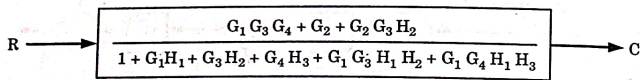


Fig. 1.9.7.

Que 1.10. For the system shown in the Fig. 1.10.1 find the overall transfer function of system using block diagram reduction.

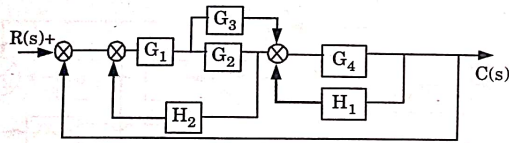


Fig. 1.10.1.

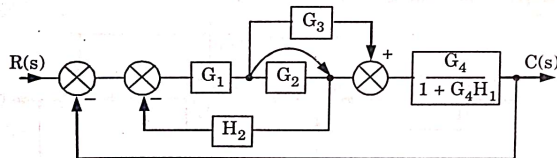
Answer

Fig. 1.10.2.

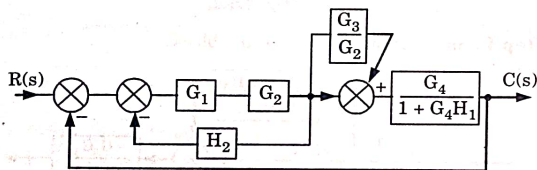


Fig. 1.10.3.

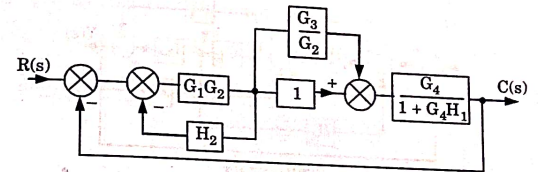


Fig. 1.10.4.

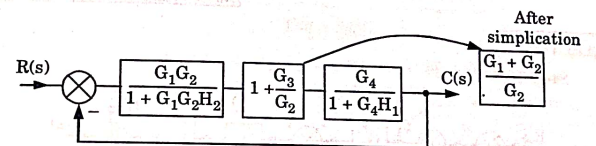


Fig. 1.10.5.

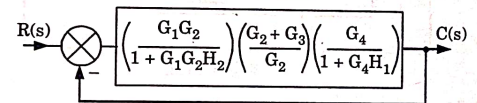


Fig. 1.10.6.

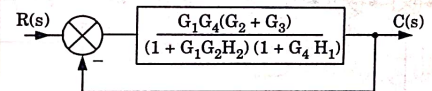


Fig. 1.10.7.

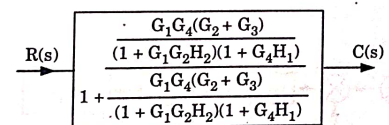


Fig. 1.10.8.

$$\frac{C(s)}{R(s)} = \frac{G_1 G_4 (G_2 + G_3)}{1 + G_1 G_2 H_2 + G_4 H_1 + G_1 G_2 G_4 H_1 H_2 + G_1 G_4 (G_2 + G_3)}$$

Que 1.11. Determine the transfer function of the given system using block diagram reduction.

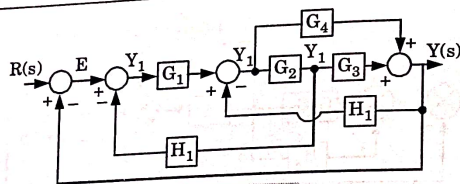


Fig. 1.11.1.

AKTU 2020-21, Marks 10

Answer

Step 1:

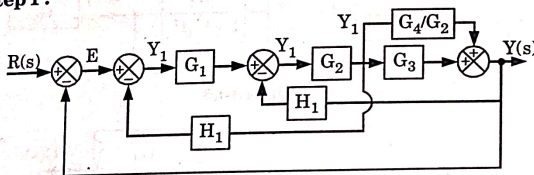


Fig. 1.11.2.

Step 2:

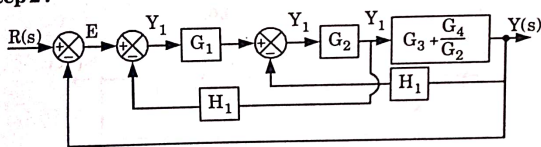


Fig. 1.11.3.

Step 3:

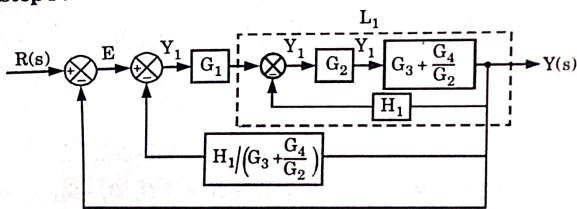


Fig. 1.11.4.

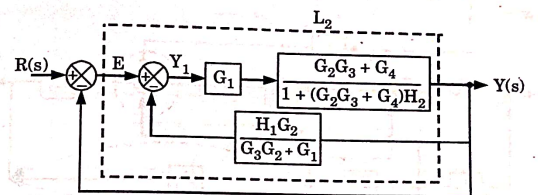
Step 4: Solving feedback loop L_1 , reduced block diagram is

Fig. 1.11.5.

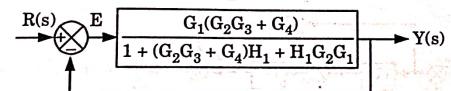
Step 5: Solving feedback loop L_2 , the reduced block diagram is

Fig. 1.11.6.

Step 6: Solving feedback loop,

$$\frac{Y(s)}{R(s)} = \frac{\frac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)H_1 + H_1G_2G_1}}{1 + \frac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)H_1 + H_1G_2G_1} \times 1}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_4H_1 + H_1G_2G_1 + G_1G_2G_3 + G_1G_4}$$

Que 1.12. Find out C/R by using block diagram reduction method.

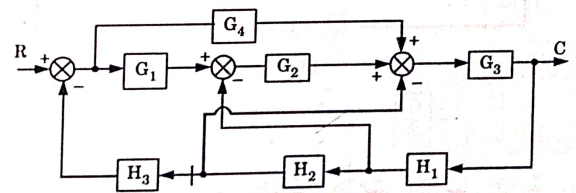


Fig. 1.12.1.

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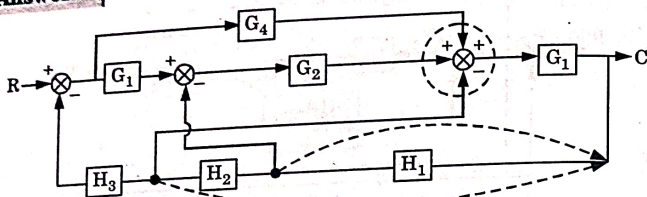
Answer

Fig. 1.12.2.

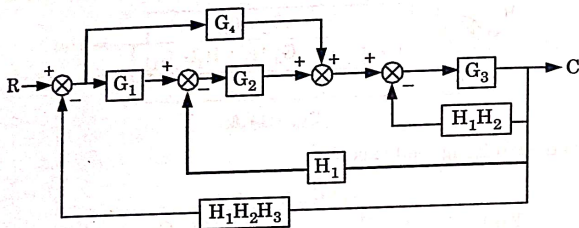


Fig. 1.12.3.

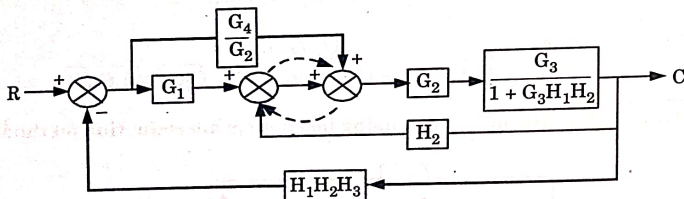


Fig. 1.12.4.

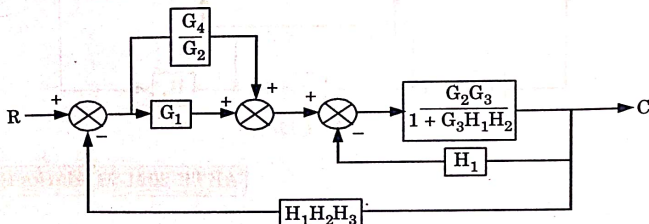


Fig. 1.12.5.

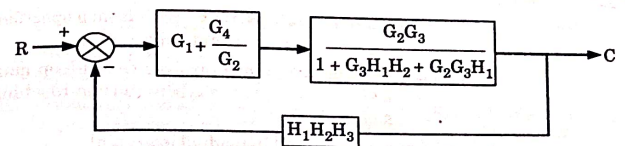


Fig. 1.12.6.

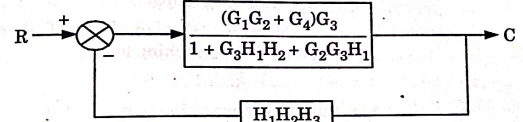


Fig. 1.12.7.

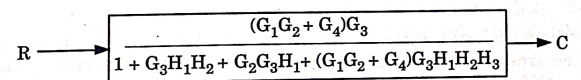


Fig. 1.12.8.

$$\frac{C}{R} = \frac{(G_1G_2 + G_4)G_3}{1 + G_3H_1H_2 + G_2G_3H_1 + (G_1G_2 + G_4)G_3H_1H_2H_3}$$

Que 1.13: Briefly explain Mason's gain formula.

OR

State and explain Masson's gain formula. For the system shown in the Fig. 1.13.1 find the overall transfer function of system using block diagram reduction.

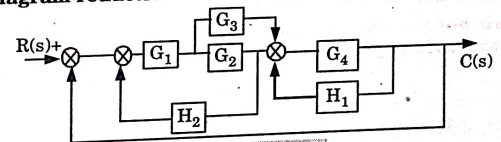


Fig. 1.13.1.

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Answer

A. Masson's gain formula :

- The overall gain can be determined by Mason's gain formula given below

$$T = \sum_{k=1}^n \frac{P_k \Delta_k}{\Delta}$$

where,

P_k = Forward path gain of k_{th} path from a specified input node to an output node.

Δ = Determinant which involves closed-loop gain and mutual interactions between non-touching loops.

$$= 1 - [\text{Sum of all individual loop gain}]$$

$$+ [\text{Sum of loop gain products of all possible pair of non-touching loops}]$$

$$- [\text{Sum of loop gain products of all possible triplets of non-touching loops}]$$

$$+ [\dots\dots\dots] - [\dots\dots\dots]$$

Δ_k = Path factor associated with the concerned path and involves all closed loops in the graph which are isolated from the forward path under consideration.

2. The path factor Δ_k for the k_{th} path is equal to the value of the graph determinant of a signal flow graph which exists after erasing the k_{th} path from the graph.

B. Numerical : Refer Q. 1.10, Page 1-13C, Unit-1.

Que 1.14. Define various terminologies used in signal flow graph.

Answer

- Source node** : The node having only outgoing branches is known as source or input node. E.g., x_0 is source node.
- Sink node** : The node having only incoming branches is known as sink or output node. E.g., x_5 is sink node.
- Chain node** : A node having incoming and outgoing branches is known as chain node. E.g., x_1, x_2, x_3 and x_4 .

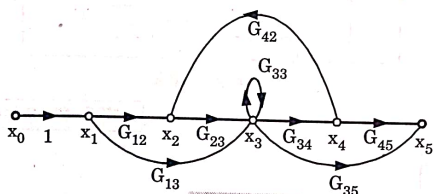


Fig. 1.14.1.

4. **Forward path** : A path from the input to output node is defined as forward path. E.g.,

$$x_0 - x_1 - x_2 - x_3 - x_4 - x_5$$

First forward path.

$$x_0 - x_1 - x_3 - x_4 - x_5$$

Second forward path.

$$x_0 - x_1 - x_3 - x_5$$

Third forward path.

$$x_0 - x_1 - x_2 - x_3 - x_5$$

Fourth forward path.

- Feedback loop** : A path which originates from a particular node and terminating at the same node, travelling through at least one other node, without tracing any node twice is called feedback loop. For example, $x_2 - x_3 - x_4 - x_2$.
- Self loop** : A feedback loop consisting of only one node is called self loop, i.e., G_{33} at x_3 is self loop.
- Path gain** : The product of branch gains while going through a forward path known as path gain. i.e. path gain for path $x_0 - x_1 - x_2 - x_3 - x_4 - x_5$ is, $1 \times G_{12} \times G_{23} \times G_{34} \times G_{45}$. This can be also called forward path gain.
- Dummy node** : If there exists incoming and outgoing branches both at first and last node representing input and output variables, then as per definition these cannot be called source and sink nodes. In such a case a separate input and output nodes can be created by adding branches with gain 1. Such nodes are called dummy nodes.

E.g.,

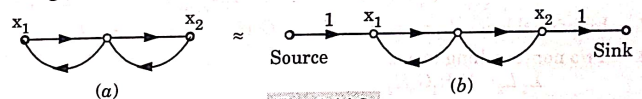


Fig. 1.14.2.

9. **Non-touching loops** : If there is no node common in between the two or more loops, such loops are said to be non-touching loops.

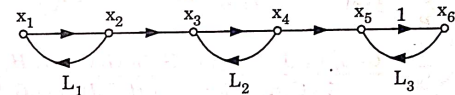


Fig. 1.14.3. Three non-touching loops.

10. **Loop gain** : The product of all the gains of the branches forming a loop is called loop gain. For a self loop, gain indicated along it is its gain. Generally such loop gain are denoted by 'L' e.g. L_1, L_2 etc.

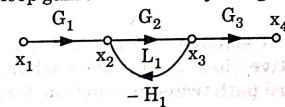


Fig. 1.14.4.

$$\text{Loop gain, } L_1 = -G_2 H_1.$$

Que 1.15. Find the overall gain of the system whose signal flow graph is shown below.

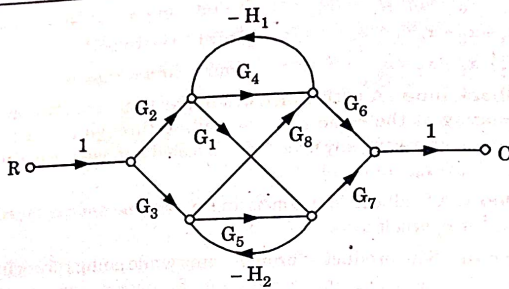


Fig. 1.15.1.

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Answer

1. The gain of the forward paths,

$$P_1 = G_2 G_4 G_6, P_2 = G_3 G_5 G_7, P_3 = G_2 G_1 G_7,$$

$$P_4 = G_3 G_8 G_6, P_5 = -G_2 G_1 H_2 G_8 G_6, P_6 = -G_3 G_8 H_1 G_1 G_7$$

2. Individual loops,
- $L_1 = -G_4 H_1$
- ,
- $L_2 = -G_5 H_2$
- ,
- $L_3 = G_1 H_2 G_8 H_1$

3. Two non-touching loops,

$$L_1 L_2 = G_4 H_1 G_5 H_2$$

$$\Delta = 1 + G_4 H_1 + G_5 H_2 - G_1 H_2 G_8 H_1 + G_4 H_1 G_5 H_2$$

$$\Delta_1 = 1 + G_5 H_2, \Delta_2 = 1 + G_4 H_1, \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

$$\begin{aligned} \frac{C}{R} &= \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4 + P_5 \Delta_5 + P_6 \Delta_6}{\Delta} \\ &= \frac{G_2 G_4 G_6 (1 + G_5 H_2) + G_3 G_5 G_7 (1 + G_4 H_1) + G_2 G_1 G_7}{1 + G_4 H_1 + G_5 H_2 - G_1 H_2 G_8 H_1 + G_4 G_5 H_1 H_2} \\ &\quad + \frac{G_3 G_8 G_6 - G_2 G_6 G_8 G_1 H_2 - G_3 G_7 G_8 G_1 H_1}{1 + G_4 H_1 + G_5 H_2 - G_1 H_2 G_8 H_1 + G_4 G_5 H_1 H_2} \end{aligned}$$

Que 1.16. Define sensitivity.

OR

Explain the effect of sensitivity on feedback system, Determine sensitivity of negative close loop system where $G(s) = 20/s(s+4)$, $H(s) = 0.5$ wrt forward path transfer function, feedback path transfer function.

AKTU 2019-20, Marks 07

Answer

- A. **Sensitivity** : Sensitivity is the change in variable due to variation in parameters of control system.

e.g.,
$$S_G^T = \frac{\% \text{ Change in } T}{\% \text{ Change in } G} = \frac{\frac{\partial T}{T} \times 100}{\frac{\partial G}{G} \times 100}$$

- B.
- Effect of sensitivity :**

$$S_G^T = \frac{\% \text{ Change in } T}{\% \text{ Change in } G} = \frac{\frac{\partial T}{T} \times 100}{\frac{\partial G}{G} \times 100}$$

$$S_G^T = \frac{G}{T} \frac{dT}{dG}$$

We know

$$T = \frac{C}{R} = \frac{G}{1+GH}$$

$$S_G^T = \left(\frac{G}{1+GH} \right) \left[\frac{(1+GH) - GH}{(1+GH)^2} \right]$$

$$S_G^T = (1+GH) \frac{1}{(1+GH)^2} = \frac{1}{1+GH}$$

So sensitivity of feedback system (with respect to G) is reduced by factor of $1+GH$.

- C.
- Numerical :**

1. Sensitivity with respect to forward path :

$$S_G^T = \frac{1}{1+GH} \quad (\text{Let } \omega = 1.5 \text{ rad/sec})$$

$$\therefore S_G^T = \frac{1}{1 + \frac{20 \times 0.5}{s(s+4)}} = \frac{s(s+4)}{s^2 + 4s + 10} = \frac{s^2 + 4s}{s^2 + 4s + 10}$$

Put $s = j\omega$,

$$S_G^T = \frac{(j\omega)^2 + 4j\omega}{(j\omega)^2 + 4(j\omega) + 10} = \frac{-2.25 + j6}{-2.25 + 10 + j6} = \frac{-2.25 + j6}{7.75 + j6}$$

$$= \frac{(-2.25 + j6) \times (-7.75 + j6)}{(j6 + 7.75)(j6 - 7.75)}$$

$$= \frac{17.43 - j60 - 36}{-36 - 60.06} = \frac{-18.57 - j60}{-96.06}$$

$$|S_G^T| = \frac{\sqrt{(18.57)^2 + (60)^2}}{96.06} = 0.65$$

2. Feedback path :

$$S_G^T = \frac{-GH}{1+GH} = \frac{-20 \times 0.5}{1 + \frac{20 \times 0.5}{s(s+4)}} = \frac{-10}{s^2 + 4s + 10}$$

Put $s = j\omega$

$$\begin{aligned} S_G^T &= \frac{-10}{(j\omega)^2 + 4j\omega + 10} = \frac{-10}{(j1.5)^2 + j4 \times 1.5 + 10} = \frac{-10}{-2.25 + j6 + 10} \\ &= \frac{-10}{7.75 + j6} \frac{(j6 - 7.75)}{(j6 - 7.75)} = \frac{-j60 + 77.5}{(j6)^2 - (7.75)^2} \\ &= \frac{\sqrt{(-60)^2 + (77.5)^2}}{96.06} = 1.02 \end{aligned}$$

Que 1.17. What are the characteristics of negative feedback ?**Answer**

The characteristics of negative feedback are as follows :

1. Accuracy in tracking steady state value.
2. Rejection of disturbance signals.
3. Low sensitivity to parameter variations.
4. Reduction in gain at the expense of better stability.

PART-5

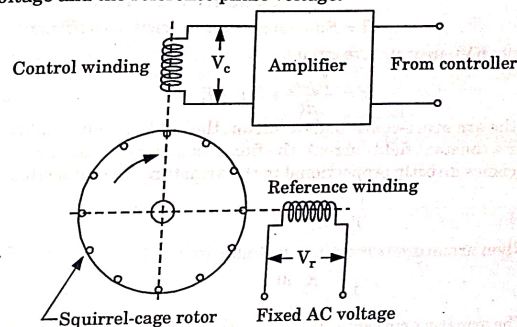
Control Systems Components : Constructional and Working Concept of AC and DC Servomotor.

Questions-Answers**Long Answer Type and Medium Answer Type Questions****Que 1.18.** Explain construction and working of AC servomotor.**Answer****A. Construction :**

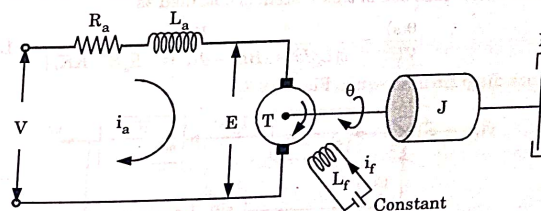
1. Fig. 1.18.1 shows the schematic diagram of a 2 ϕ AC servomotor.
2. The stator has two distributed windings which are displaced from each other by 90 electrical degrees.
3. One winding is called the reference or fixed phase and other winding is called control phase.

B. Working principle :

1. Reference phase is supplied from a constant voltage source $V_r \angle 0^\circ$. The other winding i.e., control phase is supplied with a variable voltage of the same frequency as the reference phase but its phase is displaced by 90° (electrically).
2. The control phase is usually supplied from a servo amplifier.
3. The speed and torque of the rotor are controlled by the phase difference between the control voltage and the reference phase voltages.
4. The direction of rotation of the rotor can be reversed by reversing the phase difference from leading to lagging between the control phase voltage and the reference phase voltage.

**Fig. 1.18.1.****Que 1.19.** Write a short note on armature-controlled DC servo motor.**Answer**

1. Armature controlled DC servo motor is shown in Fig. 1.19.1.

**Fig. 1.19.1.**

2. It is assumed that the demagnetizing effect of armature reaction is neglected, field voltage is constant and magnetic circuit is linear.

3. Let
- R_a = Armature resistance
 - L_a = Armature inductance
 - i_a = Armature current
 - E = Induced emf in armature
 - V = Applied armature voltage
 - θ = Angular displacement of the motor shaft
 - T = Torque developed by motor
 - J = Equivalent moment of inertia of motor shaft and load referred to the motor
 - B = Equivalent viscous friction coefficient.

4. Apply KVL in armature circuit

$$V = \frac{L_a di_a}{dt} + R_a i_a + E \quad \dots(1.19.1)$$

5. In the armature-controlled DC motor, the field current is held constant. For a constant field current, the flux becomes constant, and the torque becomes directly proportional to the armature current so that

$$T \propto \phi i_a$$

$$T = K i_a \quad \dots(1.19.2)$$

6. When armature is rotating, an emf is induced

$$E = \frac{K_b d\theta}{dt} \quad \dots(1.19.3)$$

7. The armature current produces the torque which is applied to the inertial mass and friction hence the force balance equation is

$$\frac{Jd^2\theta}{dt^2} + \frac{Bd\theta}{dt} = T = K i_a \quad \dots(1.19.4)$$

8. Taking the Laplace transform on both sides of eq. (1.19.4) and (1.19.1) ($sL_a + R_a$) $I_a(s) + E(s) = V(s)$ [Initial condition is zero]

$$(Js^2 + Bs) \theta(s) = T(s) = K i_a(s) \quad \dots(1.19.5)$$

9. The transfer function of this system is obtained as

$$\frac{\theta(s)}{V(s)} = \frac{K}{s[L_a Js^2 + (BL_a + JR_a)s + R_a B + KK_b]} \quad \dots(1.19.6)$$

10. Block diagram is shown in Fig. 1.19.2.

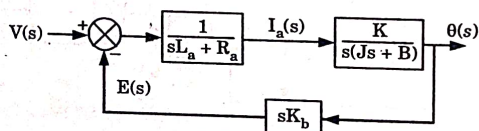


Fig. 1.19.2.

Que 1.20. Write a short note on field controlled DC servomotor.

OR

Explain the working of servomotor with suitable diagram and also derive the field controlled DC motor transfer function.

AKTU 2018-19, Marks 07

OR

Derive block diagram of armature controlled and field controlled DC motor with proper labeling of circuit diagram.

AKTU 2019-20, Marks 07

Answer

A. Working of servomotor : Refer Q. 1.18, Page 1-23C, Unit-1.

B. Armature controlled and DC motor : Refer Q. 1.19, Page 1-24C, Unit-1.

C. Field controlled DC servomotor (DC motor) :

1. A schematic diagram of a field controlled DC motor (DC servo motor) shown in Fig. 1.20.1.

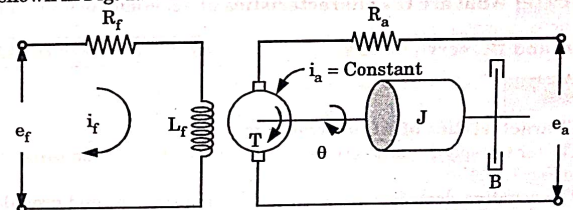


Fig. 1.20.1.

2. Here,
- R_f = Field winding resistance
 - L_f = Field winding inductance
 - I_f = Field winding current
 - R_a = Armature resistance
 - i_a = Armature current
 - θ = Angular displacement.
3. The torque T developed by the motor is proportional to product of the air-gap flux ϕ and armature current i_a so we get
- $$T = K_1 \phi i_a \quad \dots(1.20.1)$$
- where K_1 is constant.
4. But the air gap flux ϕ and the field current i_f are proportional for the usual operating range of the motor and i_a is assumed to be constant, we can rewrite the above equation as
- $$T = K_2 i_f \quad \dots(1.20.2)$$
- where K_2 is a constant.
5. The equations for this system are

$$L_f \frac{di_f}{dt} + R_f i_f = e_f \quad \dots(1.20.3)$$

$$\text{and } \frac{Jd^2\theta}{dt^2} + \frac{Bd\theta}{dt} = T = K_2 i_f \quad \dots(1.20.4)$$

6. By taking the Laplace transform on both sides of eq. (1.20.3) and (1.20.4) where all initial conditions are zero, we get

$$(L_f s + R_f) i_f(s) = E_f(s) \quad \dots(1.20.5)$$

$$(J s^2 + B s) \theta(s) = K_2 i_f(s) \quad \dots(1.20.6)$$

7. From the above equations, the transfer function of this system is obtained as

$$\frac{\theta(s)}{E_f(s)} = \frac{K_2}{s(L_f s + R_f)(J s + B)} \quad \dots(1.20.7)$$

8. Block diagram is shown in Fig. 1.20.2.

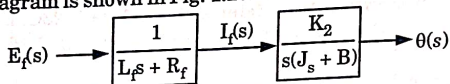


Fig. 1.20.2.

Que 1.21. What are the characteristics of servomotors? Compare

the AC and DC servomotors.

AKTU 2020-21, Marks 10

Answer

A. Characteristics of AC servomotor :

1. The torque-speed characteristics of AC servomotor (two phase) as shown in Fig. 1.21.1.
2. The negative slope represents a high rotor resistance and provides the motor with positive damping for better stability.
3. The curve is linear for almost various control voltages.

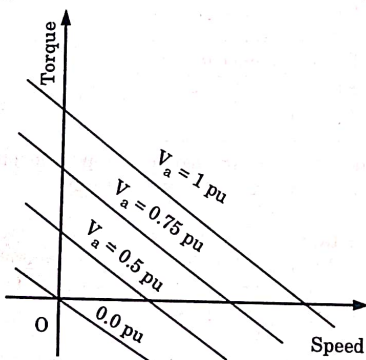


Fig. 1.21.1. Torque-speed characteristics.

B. Characteristics of DC servomotor :

1. The torque-speed characteristics of DC servomotor as shown in Fig. 1.21.2.
2. As from the characteristics, it is seen that the slope is negative. Thus, a negative slope provides viscous damping for the servo drive system.

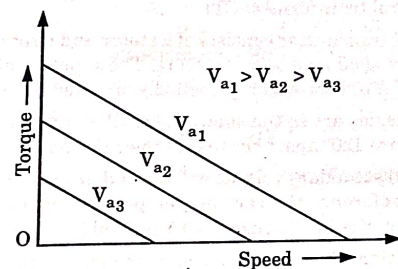


Fig. 1.21.2. Torque-speed characteristics.

B. Comparison :

S. No.	AC Servomotor	DC Servomotor
1.	It operates with low power.	It operates with higher power.
2.	It gives noiseless operation.	It gives noisy operation.
3.	Efficiency is low.	Efficiency is high.
4.	More stable.	Less stable.
5.	Less maintenance required.	More maintenance required.

PART-6

Syncho's, Stepper Motor and Tachometer.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.22. Explain principle of synchros.

Answer

1. It is a rotary transducer that converts angular displacement into an AC voltage or an AC voltage into an angular displacement.
2. A synchro system consists of
 - i. A control transmitter (CX) and
 - ii. A control transformer (CT).
3. The control transmitter consists of a stator and a rotor. The rotor is a dumb-bell shaped magnetic structure. The supply is given to the rotor by means of slip rings, which are actually mounted on the stator housing.
4. The secondaries are in the skewed slot all along the periphery of the stator and are 120° apart because of their mechanical displacement.
5. The induced secondary voltage will depend upon the angle of the rotor shaft. For reference the zero degree position of the shaft is defined when the rotor is in alignment with the coil S_2 .
6. In this position, the voltage in coil S_2 is maximum, and similarly the maximum voltage in coils S_1 and S_3 will result at 120° and 240° positions respectively. The voltage in S_2 is a function of θ and so is the voltage in S_1 and S_3 . Thus,

$$E_{0s2} = A \cos \theta$$

$$E_{0s1} = A \cos (\theta - 120^\circ)$$

$$E_{0s3} = A \cos (\theta - 240^\circ)$$

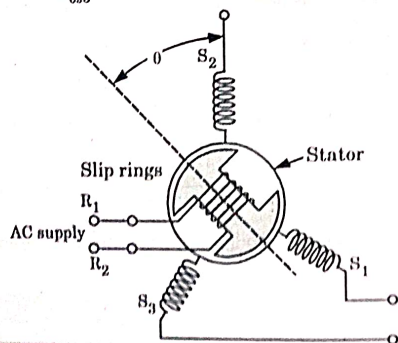


Fig. 1.22.1. Schematic diagram of synchro transmitter.

7. The connections of the synchro are made between the terminals and hence

$$\begin{aligned} E_{s1s2} &= E_{0s1} - E_{0s2} \\ &= A \cos (\theta - 120^\circ) - A \cos \theta \end{aligned}$$

$$= A \left[-\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta - \cos \theta \right]$$

$$= \sqrt{3} A \left[-\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta \right]$$

$$\begin{aligned} 8. \text{ Therefore, } E_{s2s1} &= \sqrt{3} A \left[\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta \right] \\ &= \sqrt{3} A \cos (\theta + 30^\circ) \quad \dots(1.22.1) \end{aligned}$$

$$\text{Similarly, } E_{s3s2} = \sqrt{3} A \cos (\theta + 150^\circ) \quad \dots(1.22.2)$$

$$E_{s1s3} = \sqrt{3} A \cos (\theta + 270^\circ) \quad \dots(1.22.3)$$

Que 1.23. Explain the working principle of stepper motor with neat diagram.

Answer**A. Stepper motor :**

1. A stepper motor is a form of AC motor. The input given to this motor is in the form of electric pulses.
2. For every input pulse, the motor shaft turns through a specified number of degrees, called a step.
3. Shaft of stepper motors moves through one angular step for each input pulse. The range of step size may vary from 0.72° to 90° .

B. Principle of operation :

1. Stators of stepper motors have salient poles on which concentrated windings are wound. These windings may be appropriately connected so as to result in two, three or four phase windings, on the stator.
2. The rotors of stepper motors carries no winding. The rotors are made from either permanent-magnet or ferromagnetic material.
3. A stepper motor is fed through an external drive circuit. The function of this drive circuit is to receive input voltage pulses and to deliver appropriate currents to the stator windings of the stepper motor.
4. These currents set up air-gap field which moves through one angular step for each input pulse. The rotor follows the axis of air-gap magnetic field on account of the development of reluctance torque and/or permanent-magnet torque.
5. Stepper motors vary widely in their designs and configurations, these can be classified into three main types; variables-reluctance motors, permanent-magnet motors and hybrid motors.

C. Operation of variable reluctance stepper motor :

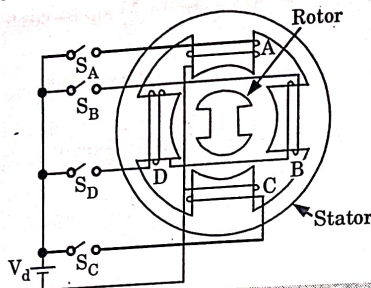


Fig. 1.23.1. 4 ϕ , 4/2 pole variable reluctance stepper motor.

1. It is a 4 ϕ , 4/2-pole (4 poles in stator and 2 in rotor), single-stack, variable reluctance stepper motor. Four phases A, B, C, and D are connected to DC source and are energized in the sequence A, B, C, D, A.
2. When winding A is excited, the rotor aligns with axis of phase A. The rotor is stable in this position and cannot move until phase A is de-energized.
3. Next phase B is excited, A is disconnected. The rotor moves through 90° in clockwise direction to align with the resultant air-gap field which now lies along the axis of phase B.
4. Further, phase C is excited and B is disconnected, the rotor moves further a step of 90° in the clockwise direction. Thus, as the phases are excited in the sequence A, B, C, D, A the rotor moves through a step of 90° at each transition in clockwise direction.
5. The rotor completes one revolution in four steps. The direction of rotation can be reversed by reversing the sequence of switching the windings, i.e., A, D, C, B, A.
6. The magnitude of step angle for any variable reluctance or permanent magnet stepper motor is given by :

$$\beta = \frac{360^\circ}{MN_r}$$

where, β = Step angle

M = Number of stator phases or stacks

N_r = Number of rotor teeth or rotor poles.

7. The step angle is also expressed as,

$$\beta = \frac{N_s - N_r}{N_s N_r} \times 360^\circ$$

where, N_s = Number of stator teeth or stator poles.

By choosing different combinations of number of rotor teeth or stator exciting coils, any desired step angle can be obtained.

Que 1.24. What is tachometer ? Also explain different types of tachometer.

Answer

A. Tachometer :

1. Tachometer is electromechanical device that convert mechanical energy into electrical energy.
2. The device works as a generator with the output voltage proportional to the magnitude of the angular velocity.

B. Types of tachometer :

a. DC tachometer :

1. The most common type of DC tachometer contains an iron-core rotor.
2. The magnetic field is provided by a permanent magnet, and no external supply voltage is necessary.
3. The windings on the rotor (armature) are connected to the commutator segments and the output voltage is taken across a pair of brushes that ride on the commutator segments.
4. The DC tachometers provide visual speed readout of a rotating shaft. Such tachometers are directly connected to a voltmeter which is calibrated in rpm.

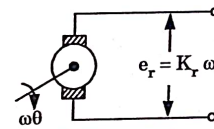


Fig. 1.24.1.

b. AC tachometer :

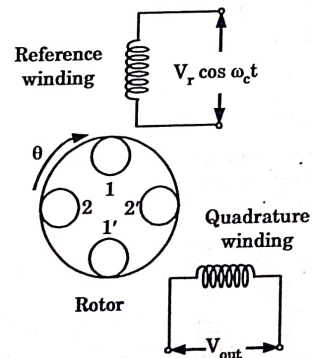


Fig. 1.24.2. AC tachometer.

1. It consists of two stator field coils or windings placed 90° electrical apart or mounted at right angles to each other. These windings are reference winding and quadrature winding.
2. A rotating element, i.e., a rotor, is placed in the air gap between the magnetic structure. Generally the rotor is a thin aluminium cup to minimize the losses and low inertia.
3. A sinusoidal voltage of rated value is applied to the primary winding. A secondary winding is placed at a 90° degree angle mechanically with respect to the primary winding.
4. When the rotor shaft is rotated, the magnitude of the sinusoidal output voltage is proportional to the rotor speed.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain open loop and closed loop control system with the help of suitable examples.

Ans. Refer Q. 1.2, Unit-1.

Q. 2. Derive force-current analogy.

Ans. Refer Q. 1.5, Unit-1.

Q. 3. Determine the transfer function of the circuit given below :

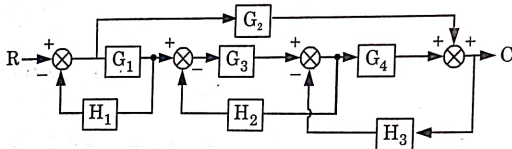
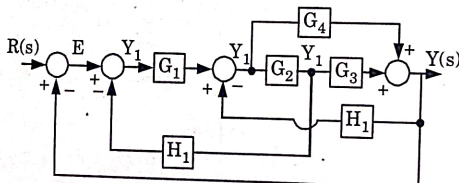


Fig. 1.

Ans. Refer Q. 1.9, Unit-1.

Q. 4. Determine the transfer function of the given system using block diagram reduction.



Ans. Refer Q. 1.11, Unit-1.

Q. 5. State and explain Masson's gain formula. For the system shown in the Fig. 1.13.1 find the overall transfer function of system using block diagram reduction.

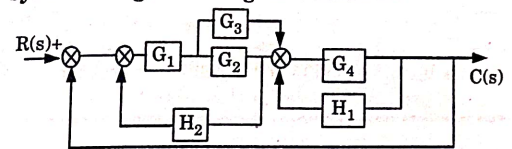


Fig. 3.

Ans. Refer Q. 1.13, Unit-1.

Q. 6. Find the overall gain of the system whose signal flow graph is shown below.

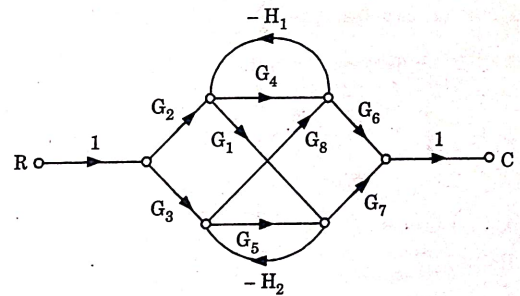


Fig. 4.

Ans. Refer Q. 1.15, Unit-1.

Q. 7. Explain construction and working of AC servomotor.

Ans. Refer Q. 1.18, Unit-1.

Q. 8. Derive block diagram of armature controlled and field controlled DC motor with proper labeling of circuit diagram.

Ans. Refer Q. 1.20, Unit-1.

Q. 9. What are the characteristics of servomotors ? Compare the AC and DC servomotors.

Ans. Refer Q. 1.21, Unit-1.



2

UNIT

Time Response Analysis

CONTENTS

Part-1 : Standard Test Signals	2-2C to 2-3C
Part-2 : Time Response Analysis of First and Second Order Systems	2-4C to 2-11C
Part-3 : Time Response Specifications of Second Order System for Unit Step Input	2-11C to 2-15C
Part-4 : Location of Roots of Characteristics Equation and Corresponding time Response	2-15C to 2-19C
Part-5 : Steady State Errors and Error Constants	2-19C to 2-27C
Part-6 : Basic Modes of Feedback Control : Proportional, Derivative, Integral and PID Controllers	2-27C to 2-33C

2-1 C (EN-Sem-5)

2-2 C (EN-Sem-5)

Time Response Analysis

PART-1

Standard Test Signals.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. Explain various standard test signals, and also find relation between them.

Answer

A. Standard test signals :

1. Unit step signal :

- Signals which start at time $t = 0$ and have magnitude of unity are called unit step signals.
- They are represented by a unit step function $u(t)$.

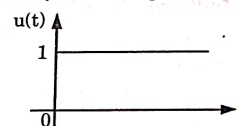


Fig. 2.1.1. Unit step.

- They are defined mathematically as :

$$u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$$

2. Unit ramp signal :

- Signals which start from zero and are linear in nature with a constant slope m are called unit ramp signals.
- They are represented by a unit ramp function $r(t)$.
- They are defined mathematically as :

$$r(t) = \begin{cases} mt; & t \geq 0 \\ 0; & t < 0 \end{cases}$$

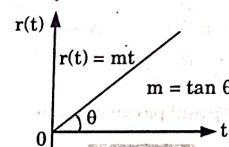


Fig. 2.1.2.

3. Unit impulse signal :

- Signals which act for very small time but have large amplitude are called unit impulse functions.
- They are represented by $\delta(t)$.
- They are defined mathematically as,

$$\delta(t) = \begin{cases} 0; & t \neq 0 \\ 1; & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

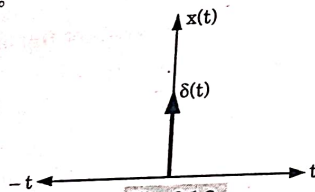


Fig. 2.1.3.

4. Unit parabolic signal : The continuous-time unit parabolic function $p(t)$, also called acceleration signal starts at $t = 0$, and is defined as :

$$p(t) = \begin{cases} \frac{t^2}{2} ; & \text{for } t \geq 0 \\ 0 ; & \text{for } t < 0 \end{cases}$$

or
$$p(t) = \frac{t^2}{2} u(t)$$

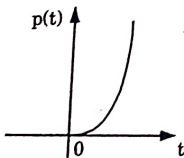


Fig. 2.1.4. Unit parabolic signal.

B. Relation between standard test signals :

- Relation between impulse and step signal :

$$\delta(t) = \frac{d}{dt} u(t)$$

- Relation between step and ramp signal :

$$u(t) = \frac{d}{dt} r(t)$$

- Relation between ramp and parabolic signal :

$$r(t) = \frac{d}{dt} \left(\frac{t^2}{2} \right)$$

PART-2

Time Response Analysis of First and Second Order Systems.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.2. Discuss the time response of first order system with unit step, unit impulse and unit ramp inputs.

OR

Derive the time response of first order system.

AKTU 2020-21, Marks 10

Answer

- Consider a first order system with unity feedback.

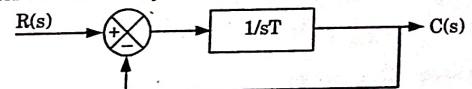


Fig. 2.2.1.

$$2. \quad \frac{C(s)}{R(s)} = \frac{\frac{1}{sT}}{\left(1 + \frac{1}{sT}\right)} = \frac{1}{(1 + sT)}$$

$$\therefore C(s) = \frac{1}{(1 + sT)} R(s) \quad \dots(2.2.1)$$

- Response to unit step input :

$$r(t) = u(t)$$

$$R(s) = \frac{1}{s}$$

Putting in eq. (2.2.1), we get

$$C(s) = \frac{1}{s} \frac{1}{(1 + sT)} = \frac{1}{s} \left(\frac{1}{s + \frac{1}{T}} \right)$$

Taking inverse Laplace transform, we have

$$L^{-1}[C(s)] = c(t) = L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left[\frac{1}{\left(s + \frac{1}{T}\right)}\right]$$

4. Response to unit impulse input :

$$c(t) = 1 - e^{-t/T}$$

$$r(t) = \delta(t)$$

$$R(s) = 1$$

Putting in eq. (2.2.1),

$$C(s) = \frac{1}{(sT+1)} = \frac{1}{T} \frac{1}{\left(s + \frac{1}{T}\right)}$$

Taking inverse Laplace transform, we have

$$L^{-1}[C(s)] = \frac{1}{T} L^{-1}\left[\frac{1}{\left(s + \frac{1}{T}\right)}\right]$$

$$c(t) = \frac{1}{T} e^{-t/T}$$

5. Response to unit ramp input :

$$r(t) = t$$

$$R(s) = \frac{1}{s^2}$$

Putting in eq. (2.2.1), we get

$$\begin{aligned} C(s) &= R(s) \frac{1}{(sT+1)} = \frac{1}{s^2(sT+1)} \\ &= \frac{1}{s^2} - \frac{T}{s} + \frac{T}{\left(s + \frac{1}{T}\right)} \end{aligned}$$

Taking inverse Laplace transform, we have

$$L^{-1}[C(s)] = L^{-1}\left(\frac{1}{s^2}\right) - TL^{-1}\left(\frac{1}{s}\right) + TL^{-1}\left[\frac{1}{\left(s + \frac{1}{T}\right)}\right]$$

$$c(t) = t - T + Te^{-t/T}$$

Que 2.3. The impulse response of unity feedback closed loop system is, $c(t) = -te^{-t} + 2e^{-t}$, find the open loop transfer function.

Answer

Given : $r(t) = \delta(t)$, $c(t) = -te^{-t} + 2e^{-t}$, $H(s) = 1$
To Find : Transfer function.

1. $r(t) = \delta(t)$
2. Laplace transform of $r(t)$ gives,

$$R(s) = 1$$

$$c(t) = -te^{-t} + 2e^{-t}$$

3. Laplace transform of $c(t)$ gives

$$C(s) = \frac{-1}{(s+1)^2} + \frac{2}{(s+1)}$$

$$C(s) = \frac{-1+2(s+1)}{(s+1)^2} = \frac{(2s+1)}{(s+1)^2} \quad \dots(2.3.1)$$

5. For unity feedback control system

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$C(s) = \frac{G(s)}{1+G(s)} R(s) \quad \dots(2.3.2)$$

6. Putting value of $C(s)$ and $R(s)$ in eq. (2.3.2)

$$\frac{(2s+1)}{(s+1)^2} (1+G(s)) = G(s) \times 1$$

$$\begin{aligned} \frac{(2s+1)}{(s+1)^2} &= G(s) \left[1 - \frac{(2s+1)}{(s+1)^2} \right] \\ &= G(s) \left[\frac{s^2+1+2s-2s-1}{(s+1)^2} \right] \end{aligned}$$

7. Open loop transfer function is

$$\therefore G(s) = \frac{(2s+1)}{s^2}$$

Que 2.4. Derive the expression for the time response of a second order control system subjected to a unit step input.

OR

Derive the expressions for second order system for underdamped case and when the input is unit step.

AKTU 2018-19, Marks 07

Answer

- A. Response of a second order system for unit step input :**

1. Consider the second order system with unity feedback.
2. The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2 / s(s + 2\xi\omega_n)}{1 + \omega_n^2 / s(s + 2\xi\omega_n)}$$

where,

ξ = Damping factor or Damping ratio

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

Then output, $C(s) = R(s) \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$... (2.4.1)

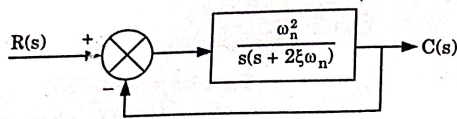


Fig. 2.4.1.

3. For unit step input

$$r(t) = 1$$

$$R(s) = \frac{1}{s}$$

Then $C(s) = \frac{1}{s} \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$... (2.4.2)

4. In eq. (2.4.2) putting $[s^2 + 2\xi\omega_n s + \omega_n^2] = [(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)]$ and breaking it into partial fraction

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{[(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)]} \quad \dots (2.4.3)$$

Put

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} \quad \dots (2.4.4)$$

5. Rewrite eq. (2.4.4)

$$C(s) = \frac{1}{s} - \frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n}{\omega_d} \frac{\omega_d}{(s + \xi\omega_n)^2 + \omega_d^2} \quad \dots (2.4.5)$$

6. Taking inverse Laplace transform of eq. (2.4.5),

$$c(t) = 1 - e^{-\xi\omega_n t} \cos \omega_d t - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t} \sin \omega_d t$$

Put

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} [(\sqrt{1 - \xi^2}) \cos \omega_d t + \xi \sin \omega_d t] \quad \dots (2.4.6)$$

7. Put

$$\sin \phi = \sqrt{1 - \xi^2},$$

$\therefore$

$$\cos \phi = \xi$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} (\sin \phi \cos \omega_d t + \cos \phi \sin \omega_d t)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \phi) \quad \dots (2.4.7)$$

where

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

and

$$\phi = \tan^{-1} \left(\frac{\sqrt{1 - \xi^2}}{\xi} \right)$$

8. Eq. (2.4.7) is rewritten as

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} \sin \left[(\omega_n \sqrt{1 - \xi^2}) t + \tan^{-1} \left(\frac{\sqrt{1 - \xi^2}}{\xi} \right) \right] \quad \dots (2.4.8)$$

9. The term ω_n is called natural frequency of oscillations. Term $\omega_d = \omega_n \sqrt{1 - \xi^2}$ is called damped frequency of oscillations and the term ξ is called damping ratio or damping factor.

B. Response of second order systems for different cases :

1. **Underdamped case ($0 < \xi < 1$) :** From eq. (2.4.8), time constant is $1/\xi\omega_n$ and the response having damped oscillations with overshoot and undershoot is known as underdamped response.

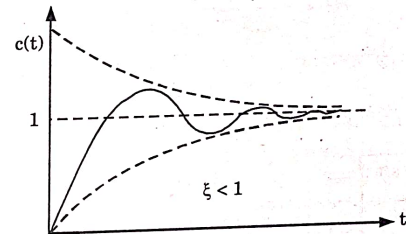


Fig. 2.4.2. Underdamped oscillations.

2. **Undamped case ($\xi = 0$) :**

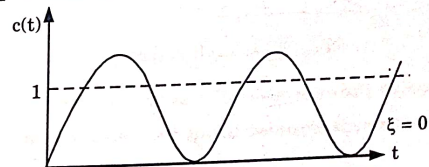


Fig. 2.4.3. Undamped oscillations.

Put

$$\xi = 0$$

Eq. (2.4.8) becomes,

$$c(t) = 1 - \sin(\omega_n t + \pi/2)$$

$$c(t) = 1 - \cos \omega_n t$$

3 Critically damped case ($\xi = 1$):

At

$$\xi = 1$$

$$c(t) = 1 + e^{-\xi \omega_n t} (1 + \omega_n t)$$

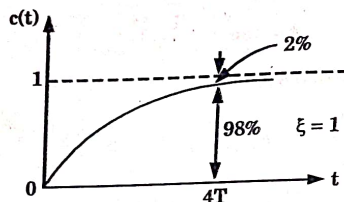


Fig. 2.4.4. Critically damped.

4 Overdamped case ($\xi > 1$):

$$C(s) = \frac{\omega_n^2}{(s + \xi\omega_n + \omega_n\sqrt{\xi^2 - 1})(s + \xi\omega_n - \omega_n\sqrt{\xi^2 - 1})}$$

Taking inverse Laplace both side,

$$c(t) = 1 + \frac{e^{-(\xi - \sqrt{\xi^2 - 1})\omega_n t}}{2\sqrt{\xi^2 - 1}(\xi + \sqrt{\xi^2 - 1})} - \frac{e^{-(\xi + \sqrt{\xi^2 - 1})\omega_n t}}{2\sqrt{\xi^2 - 1}(\xi - \sqrt{\xi^2 - 1})}$$

neglect

$$c(t) = 1 - \frac{e^{-(\xi - \sqrt{\xi^2 - 1})\omega_n t}}{2\sqrt{\xi^2 - 1}(\xi - \sqrt{\xi^2 - 1})}$$

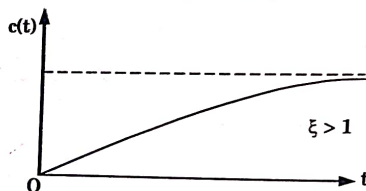


Fig. 2.4.5. Overdamped.

Que 2.5. Derive the expression for second order system response when subjected to unit impulse input for damping ratio ($\xi < 1$).

Answer

1. For unit impulse function, $R(s) = 1$ and the output of a second order system is given by

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \dots(2.5.1)$$

2. In eq. (2.5.1) rewriting the term $(s^2 + 2\xi\omega_n s + \omega_n^2)$ as $[(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)]$

$$C(s) = \frac{\omega_n^2}{[(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)]}$$

$$C(s) = \frac{\omega_n^2}{\omega_n\sqrt{1 - \xi^2}} \frac{\omega_n\sqrt{1 - \xi^2}}{[(s + \xi\omega_n)^2 + (\omega_n\sqrt{1 - \xi^2})^2]} \quad \dots(2.5.2)$$

3. Taking inverse Laplace transform on both sides of eq. (2.5.2),

$$c(t) = \frac{\omega_n}{\sqrt{1 - \xi^2}} e^{-\xi\omega_n t} \sin[(\omega_n\sqrt{1 - \xi^2})t] \quad \dots(2.5.3)$$

4. For $\xi < 1$,

$$c(t) = \frac{\omega_n}{\sqrt{1 - \xi^2}} e^{-\xi\omega_n t} \sin[(\omega_n\sqrt{1 - \xi^2})t] \quad \dots(2.5.4)$$

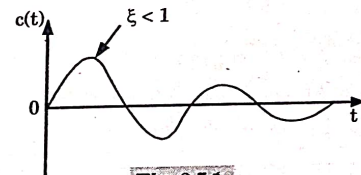


Fig. 2.5.1.

5. The time response for $\xi < 1$ is decaying exponential oscillations and the output at times goes negative also.

Que 2.6. The open loop transfer function of a unity feedback control system is given by $G(s) = 9/s(s+3)$. Find the natural frequency of response, damping ratio, damped frequency and time constant.

AKTU 2020-21, Marks 10

Answer

Given: $G(s) = \frac{9}{s(s+3)}$, $H(s) = 1$

To Find: ω_n , ξ , ω_d , T .

1. Transfer function of closed loop system,

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{\frac{9}{s(s+3)}}{1 + \frac{9}{s(s+3)} \times 1} = \frac{9}{s^2 + 3s + 9} \quad \dots(2.6.1)$$

2. Comparing eq. (2.6.1) by standard second order characteristic equation,

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n^2 = 9$$

3. Natural frequency, $\omega_n = 3$ rad/sec

$$2\xi\omega_n = 3$$

$$2 \times \xi \times 3 = 3$$

4. Damping ratio, $\xi = 0.5$

5. Damped frequency,

$$\omega_d = \omega_n \sqrt{1 - \xi^2} = 3\sqrt{1 - (0.5)^2} = 2.59 \text{ rad/sec}$$

6. Time constant, $T = \frac{1}{\xi\omega_n} = \frac{1}{0.5 \times 3} = 0.66 \text{ sec}$

PART-3

Time Response Specifications of Second Order System for Unit Step Input.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.7. Draw time domain response curve of a second order system and indicate important specifications.
OR

Define the following term :

- Rise time
- Peak time
- Peak overshoot
- Settling time

AKTU 2020-21, Marks 10

Answer

1. **Delay time (t_d)** : It is the time required for the response to reach 50 % of the final value in first time.

$$t_d = \frac{1 + 0.7\xi}{\omega_n}$$

2. **Rise time (t_r)** : It is the time required for the response to rise from 10 % to 90 % of its final value for overdamped system and 0 to 100 % for underdamped systems.

$$t_r = \frac{\pi - \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n \sqrt{1-\xi^2}}$$

3. **Peak time (t_p)** : The peak time is the time required for the response to reach the first peak of the time response or first peak overshoot.

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

4. **Maximum overshoot (M_p)** : It is the normalized difference between the peak of the time response and steady output. The maximum percent overshoot is defined as

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$$\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$

5. **Settling time (t_s)** : The settling time is the time required for the response to reach and stay within the specified range (2 % to 5 %) of its final value.

$$t_s = \frac{4}{\xi\omega_n}$$

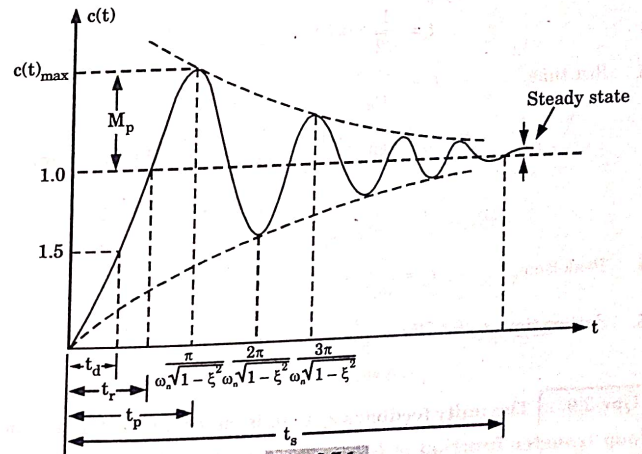


Fig. 2.7.1.

$$t_r = \frac{\pi - 0.92}{13.32} = 0.16 \text{ sec}$$

8. Peak time, $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{16.66\sqrt{1-0.6^2}} = 0.23 \text{ sec}$

9. Delay time, $t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7 \times 0.6}{16.66} = 0.085 \text{ sec}$

PART-4

Location of Roots of Characteristics Equation and Corresponding time Response.

Questions-Answers**Long Answer Type and Medium Answer Type Questions**

Que 2.10. Draw the root locus of characteristics equation for second order system as damping ratio varies from $-\infty$ to $+\infty$ keeping W_n constant. **AKTU 2019-20, Marks 3.5**

OR

Draw the roots of the characteristics equation for second order system and corresponding time response.

Answer

The effect of roots of the characteristic equation on the damping ratio of a second order system is shown in Fig. 2.10.1 and Fig. 2.10.2.

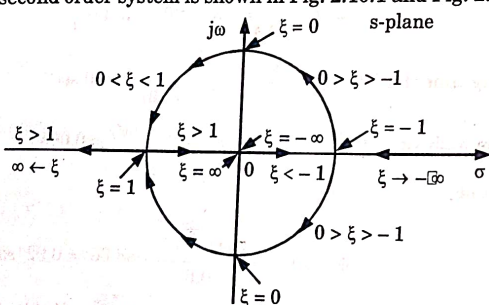


Fig. 2.10.1. Locus of roots of the characteristic equation of the prototype second-order system, when W_n is held constant while the damping ratio is varied from $-\infty$ to $+\infty$.

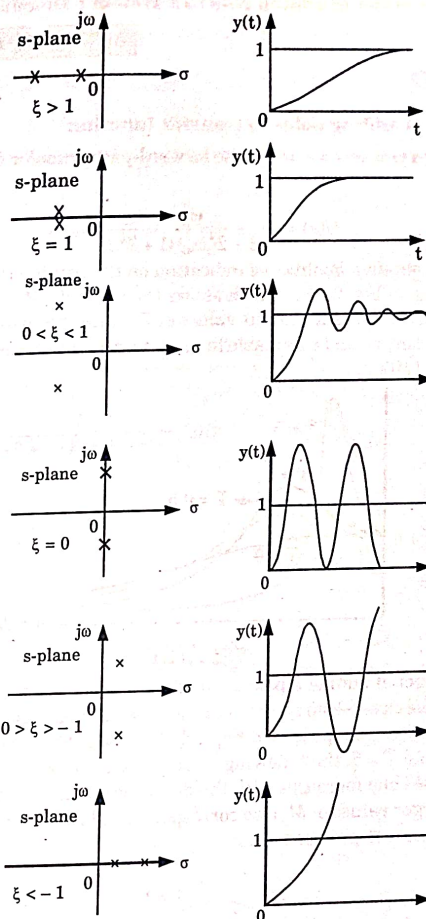


Fig. 2.10.2. Step-response comparison for various characteristic-equation-root locations in the s-plane.

Que 2.11. Explain the effect of adding poles and zero to transfer function. **AKTU 2019-20, Marks 3.5**

OR

What is the effect of adding pole to a system ? Discuss.

AKTU 2018-19, Marks 07

Answer

A. Effect of adding poles to transfer function :

- Adding a pole at $s = -1/T$ to the forward-path transfer function of lead to

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)(1 + Ts)}$$

- We can obtain a qualitative indication on the bandwidth properties by referring to Fig. 2.11.1, which shows the plots of $|M(j\omega)|$ versus ω for $\omega_n = 1$, $\xi = 0.707$, and various values of T . Since the system is now of the third order, it can be unstable for a certain set of system parameters.

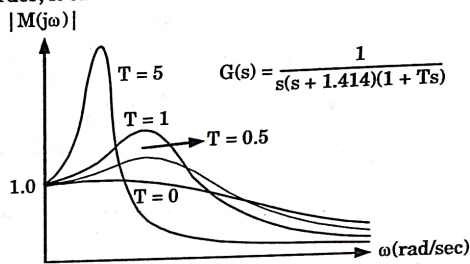


Fig. 2.11.1.

- The effect of adding a pole to the forward-path transfer function is to make the closed-loop system less stable, while decreasing the bandwidth.
- The unit-step responses of Fig. 2.11.2 show that for larger values of T , $T = 1$ and $T = 5$, the following relations are observed :
 - The rise time increases with the decrease of the bandwidth,
 - The larger values of M_r also correspond to a larger maximum overshoot in the unit-step responses.

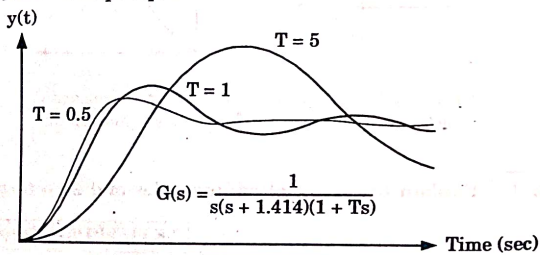


Fig. 2.11.2.

B. Effect of adding a zero to transfer function :

- Consider unity feedback system having forward path transfer function as,

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)} \quad \dots(2.11.1)$$

- Let us add a zero at $s = -1/T$ to the transfer function so that eq. (2.11.1) becomes

$$G(s) = \frac{\omega_n^2(1 + Ts)}{s(s + 2\xi\omega_n)}$$

- The closed-loop transfer function for unity feedback system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2(1 + Ts)}{s^2 + (2\xi\omega_n + T\omega_n^2)s + \omega_n^2}$$

$$\text{B.W.} = \sqrt{\left(-b + \frac{1}{2}\sqrt{b^2 + 4\omega_n^4}\right)}$$

where
for

$$b = 4\xi^2\omega_n^2 + 4\xi\omega_n^3 + T - 2\omega_n^2 - \omega_n^4$$

$$\xi = 0.707 \text{ and } \omega_n = 1$$

B.W

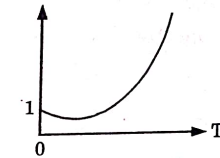


Fig. 2.11.3.

- The general effect of adding a zero to the forward path transfer function is to increase the bandwidth of the closed-loop system.

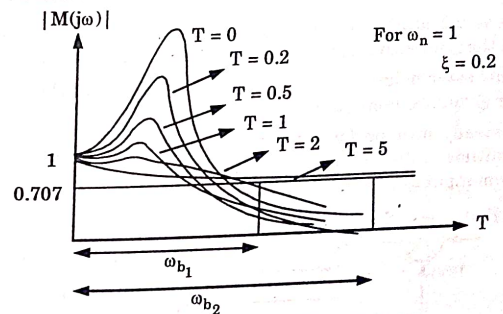


Fig. 2.11.4.

- So bandwidth increases as T -increases

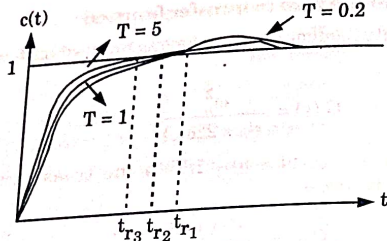


Fig. 2.11.5.

$t_{r3} < t_{r2} < t_{r1}$
So as T increases rise time decreases..

PART-5**Steady State Errors and Error Constants.****Questions-Answers****Long Answer Type and Medium Answer Type Questions**

Que 2.12. Discuss steady state error.

Answer

1. The difference between the steady-state response and desired reference give steady state error.
2. If the actual output of control system during steady state error deviates from the reference input, then system possess steady state error.
3. Steady error helps in determining the accuracy, so the steady state error should be minimum.
4. The steady state performance of a control system is assessed by the magnitude of the steady state error possessed by the system and the system input can be step or ramp or parabolic.

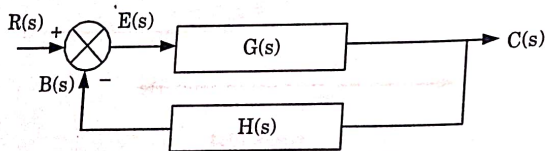


Fig. 2.12.1. Canonical diagram.

5. The magnitude of steady state error in a closed loop control system depends on its open loop control function. i.e., $G(s)H(s)$ of the system.

$$E(s) = R(s) - B(s)$$

6. But

$$B(s) = C(s)H(s)$$

$$\therefore E(s) = R(s) - C(s)H(s)$$

Also

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - G(s)E(s)H(s)$$

$$E(s)[1 + G(s)H(s)] = R(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

...(2.12.1)

If positive feedback is used, the expression will be

$$E(s) = \frac{R(s)}{1 - G(s)H(s)}$$

...(2.12.2)

8. Combining eq. (2.12.1), and (2.12.2)

$$E(s) = \frac{R(s)}{1 \pm G(s)H(s)}$$

...(2.12.3)

For a unity feedback system $H(s) = 1$

$$E(s) = \frac{R(s)}{1 \pm G(s)}$$

...(2.12.4)

9. $\therefore$
10. In time domain $e(t) = L^{-1}[E(s)]$. It is the expression of error valid for all time. Steady state error is defined as :

$$e_{ss} = \lim_{t \rightarrow \infty} e(t)$$

11. Using final value theorem

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} \frac{sR(s)}{1 \pm G(s)H(s)}$$

Que 2.13. Derive an expression for K_p , K_v and K_a for type-0, type-1 and type-2 system.

OR

Derive the expression for steady state error and explain K_p , K_v and K_a .

AKTU 2020-21, Marks 10

Answer

- A. Expression for steady state error : Refer Q. 2.12, Page 2-19C, Unit-2.

- B. Expression for K_p , K_v and K_a :

- i. Unit step input :

$$\text{Here } r(t) = \begin{cases} 1; & t > 0 \\ 0; & t < 0 \end{cases}$$

$$R(s) = \frac{1}{s}$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)} \quad \dots(2.13.1)$$

$$E(s) = \frac{1}{s + sG(s)H(s)}$$

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \frac{1}{s + sG(s)H(s)}$$

$$= \frac{1}{1 + \lim_{s \rightarrow 0} G(s)H(s)} = \frac{1}{1 + K_p}$$

where,

$$K_p = \lim_{s \rightarrow 0} G(s)H(s)$$

K_p = Position error constant.

For step input, $e_{ss} = \frac{1}{1 + K_p}$

Case 1 : For type '0'

$$K_p = \text{Constant}$$

$$e_{ss} = \text{Constant}$$

Case 2 : For type '1'

$$K_p = \infty$$

$$e_{ss} = \frac{1}{1 + \infty} = 0$$

Case 3 : For type '2'

$$K_p = \infty$$

$$e_{ss} = \frac{1}{1 + \infty} = 0$$

ii. Ramp input :

Here

$$r(t) = \begin{cases} t & ; t > 0 \\ 0 & ; t < 0 \end{cases}$$

$$R(s) = \frac{1}{s^2}$$

$$E(s) = \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$= \frac{1}{\lim_{s \rightarrow 0} sG(s)H(s)}$$

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s \rightarrow 0} sG(s)H(s)$$

K_v = Velocity error constant.

Case 1 : For type '0'

$$K_v = \lim_{s \rightarrow 0} sG(s)H(s) = 0$$

$$e_{ss} = \frac{1}{K_v} = \infty$$

Case 2 : For type '1'

$$K_v = \text{constant}$$

$$e_{ss} = \frac{1}{K_v} = \text{constant}$$

Case 3 : For type '2'

$$K_v = \infty$$

$$e_{ss} = 0$$

iii. Parabolic input :

Here

$$r(t) = \begin{cases} (1/2)t^2 & ; t > 0 \\ 0 & ; t < 0 \end{cases}$$

$$R(s) = \frac{1}{s^3}$$

$$E(s) = \frac{1}{s^3 + s^3 G(s)H(s)}$$

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \rightarrow 0} s^2 G(s)H(s)} = \frac{1}{K_a}$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)H(s)$$

K_a = Acceleration error constant.

Case 1 : For type '0'

$$K_a = 0$$

$$e_{ss} = \frac{1}{0} = \infty$$

Case 2 : For type '1'

$$K_a = 0$$

$$e_{ss} = \frac{1}{0} = \infty$$

Case 3 : For type '2'

$$K_a = \text{Constant}$$

$$e_{ss} = \text{Constant}$$

Que 2.14. For the unity feedback control system the open loop transfer function, $G(s) = 10(s+2)/s^2(s+4)$. Find the e_{ss} when the input is $r(t) = 3 - 2t + 3t^2$. And find K_p , K_v and K_a .

AKTU 2020-21, Marks 10

Answer

Given : $G(s) = \frac{10(s+2)}{s^2(s+4)}$, $H(s) = 1$, $r(t) = 3 - 2t + 3t^2$

To Find : e_{ss} , K_p , K_v and K_a .

- $K_p = \lim_{s \rightarrow 0} G(s) H(s) = \lim_{s \rightarrow 0} \frac{10(s+2)}{s^2(s+4)} = \infty$
- $K_v = \lim_{s \rightarrow 0} sG(s) H(s) = \lim_{s \rightarrow 0} \frac{s \cdot 10(s+2)}{s^2(s+4)} = \infty$
- $K_a = \lim_{s \rightarrow 0} s^2 G(s) H(s) = \lim_{s \rightarrow 0} \frac{s^2 \cdot 10(s+2)}{s^2(s+4)} = 5$
- Now $r(t) = 3 - 2t + 3t^2$
 $R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{3}{s^3}$
 $R_1 = 3$ (Constant)
 $R_2 = -2$ (Co-efficiency of t)
 $R_3 = 3$ (Co-efficiency of $t^2/2$)
- $e_{ss} = \frac{R_1}{1+K_p} + \frac{R_2}{K_v} + \frac{R_3}{K_a}$
 $= \frac{3}{1+\infty} + \frac{(-2)}{\infty} + \frac{3}{5} = 0.6$

Que 2.15. Find out various error coefficients : K_p , K_v , K_a and steady state error for standard step, ramp, and parabolic inputs for system shown below :

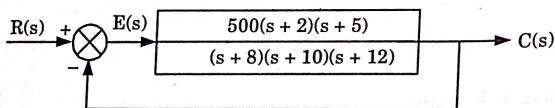


Fig. 2.15.1.

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Answer

Given : $G(s) = \frac{500(s+2)(s+5)}{(s+8)(s+10)(s+12)}$, $H(s) = 1$

To Find : K_p , K_v , K_a and e_{ss} .

- $K_p = \lim_{s \rightarrow 0} G(s) H(s) = \lim_{s \rightarrow 0} \frac{500(s+2)(s+5)}{(s+8)(s+10)(s+12)} = 5.20$
- $K_v = \lim_{s \rightarrow 0} sG(s) H(s) = \lim_{s \rightarrow 0} \frac{s \times 500(s+2)(s+5)}{(s+8)(s+10)(s+12)} = 0$
- $K_a = \lim_{s \rightarrow 0} s^2 G(s) H(s) = \lim_{s \rightarrow 0} \frac{s^2 \times 500(s+2)(s+5)}{(s+8)(s+10)(s+12)} = 0$
- Steady state error for step input,
 $e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+5.20} = 0.16$
- Steady state error for ramp input,
 $e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$
- Steady state error for parabolic input,
 $e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$

Que 2.16. Find the generalized error coefficients for a system whose $G(s) H(s) = 1/s(s+2)$ and also find the expression for steady state error for input $r(t) = 2 + 3t + 2t^3$. AKTU 2018-19, Marks 07

Answer

- $r(t) = 2 + 3t + 2t^3$; $r'(t) = 3 + 6t^2$;
 $r''(t) = 12t$; $r'''(t) = 12$; $r^{(4)}(t) = 0$.
 The coefficients C_0 , C_1 , C_2 and C_3 need to be determined.
- $\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{1}{1 + \frac{1}{s(s+2)}} = \frac{s^2 + 2s}{s^2 + 2s + 1} = \frac{s^2 + 2s}{(s+1)^2}$
- $E(s) = \frac{s^2 + 2s}{(s+1)^2} R(s) \therefore F_e(s) = \frac{s^2 + 2s}{(s+1)^2}$
- $C_0 = \lim_{s \rightarrow 0} F_e(s) = \lim_{s \rightarrow 0} \frac{s^2 + 2s}{(s+1)^2} = 0$

5.

$$C_1 = \lim_{s \rightarrow 0} \frac{dF_e(s)}{ds} = \lim_{s \rightarrow 0} \frac{d}{ds} \left\{ \frac{s^2 + 2s}{(s+1)^2} \right\}$$

$$= \lim_{s \rightarrow 0} \frac{(s+1)^2(2s+2) - (s^2+2s)(2)(s+1)(1)}{(s+1)^4}$$

$$= \lim_{s \rightarrow 0} \frac{2}{(s+1)^3} = 2$$

6.

$$C_2 = \lim_{s \rightarrow 0} \frac{d^2 F_e(s)}{ds^2} = \lim_{s \rightarrow 0} \frac{d}{ds} \left\{ \frac{dF_e(s)}{ds} \right\}$$

$$= \lim_{s \rightarrow 0} \frac{d}{ds} \left\{ \frac{2}{(s+1)^3} \right\}$$

$$= \lim_{s \rightarrow 0} \frac{(s+1)^3(0) - 2(3)(s+1)^2(1)}{(s+1)^6}$$

$$= \lim_{s \rightarrow 0} -\frac{6}{(s+1)^4} = -6$$

7.

$$C_3 = \lim_{s \rightarrow 0} \frac{d^3 F_e(s)}{ds^3} = \lim_{s \rightarrow 0} \frac{d}{ds} \left\{ \frac{d^2 F_e(s)}{ds^2} \right\}$$

$$= \lim_{s \rightarrow 0} \frac{d}{ds} \left\{ -\frac{6}{(s+1)^4} \right\}$$

$$= \lim_{s \rightarrow 0} \left\{ \frac{(s+1)^4(0) - 6 \times 4(s+1)^3(1)}{(s+1)^8} \right\}$$

$$= \lim_{s \rightarrow 0} \frac{24}{(s+1)^5} = 24$$

Que 2.17. The close loop transfer function is given by

$$T(s) = k(s+z)/(s^2+4s+8), \text{ Where } k, z \text{ is adjustable.}$$

- i. If $r(t) = t$ finds k and z so that steady error is zero.
 ii. For the value of k, z obtain in part (i) find $e(\infty)$ for input $r(t) = t^2/2$

Answer

i.

1. Given, $\frac{C(s)}{R(s)} = \frac{k(s+z)}{s^2+4s+8}$

and,

$$e_{ss} = 0$$

2. We know

$$e_{ss} = \lim_{s \rightarrow 0} sE(s)$$

$$0 = \lim_{s \rightarrow 0} sE(s)$$

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but
 then

$$E(s) = R(s) - C(s)$$

$$0 = \lim_{s \rightarrow 0} s [R(s) - C(s)]$$

or

$$R(s) = C(s)$$

$$R(s) = \frac{k(s+z)}{s^2+4s+8}$$

$$s^2 + 4s + 8 = k(s+z)$$

3. Compare the coefficients of s in the both sides.

$$s^1 \text{ coefficient} \quad 4 = k$$

$$s^0 \text{ coefficient} \quad 8 = kz$$

or,

$$z = 2$$

ii.

1. Given,

$$r(t) = t^2/2$$

2.

Taking Laplace transform, $R(s) = -1/s^3$

3.

We know

$$e_{ss} = \lim_{s \rightarrow 0} sE(s)$$

$$E(s) = R(s) - C(s) = R(s) - \frac{k(s+z)}{(s^2+4s+8)} R(s)$$

4. Putting the value of k and z then we get,

$$E(s) = R(s) - \frac{4(s+2)}{(s^2+4s+8)} R(s)$$

$$E(s) = \frac{(s^2+4s+8-4s-8)}{s^2+4s+8} R(s)$$

$$e_{ss} = \left| \lim_{s \rightarrow 0} s \frac{s^2}{s^2+4s+8} \times -\frac{1}{s^3} \right| = \frac{1}{8} = 0.125$$

Que 2.18. Explain steady state error due to step input for type 0, 1 and 2 systems. Find steady state error for $G(s) = 10(1+4s)/s^2(1+s)$, $H(s) = 1$, input $r(t) = 1 + t + t^2/2$.

AKTU 2019-20, Marks 07**Answer**

A. Steady state error : Refer Q. 2.13, Page 2-20C, Unit-2.

B. Numerical : $G(s) = \frac{10(1+4s)}{s^2(1+s)}$

$$H(s) = 1$$

1.

$$K_p = \lim_{s \rightarrow 0} G(s) H(s) = \lim_{s \rightarrow 0} \frac{10(1+4s)}{s^2(1+s)} = \infty$$

2.

$$K_v = \lim_{s \rightarrow 0} s G(s) H(s) = \lim_{s \rightarrow 0} s \frac{10(1+4s)}{s^2(1+s)} = \infty$$

3.

$$K_a = \lim_{s \rightarrow 0} s^2 G(s) H(s) = \lim_{s \rightarrow 0} \frac{10(1+4s)}{(1+s)} = 10$$

$$r(t) = 1 + t + \frac{t^2}{2}$$

4. Steady state error is given by,

$$e_{ss} = \lim_{s \rightarrow 0} s \frac{R(s)}{1 + G(s)H(s)}$$

$$e_{ss} = \frac{R_1}{1 + K_p} + \frac{R_2}{K_v} + \frac{R_3}{K_a} = \frac{1}{1 + \infty} + \frac{1}{\infty} + \frac{1/2}{10} = 0.05$$

PART-6

Basic Modes of Feedback Control : Proportional, Derivative, Integral and PID Controllers.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.19. Write a short note on proportional control. Also write its advantages and disadvantages.

Answer

A. Proportional control :

1. In proportional control the actuating signal for the control action in a control system is proportional to the error signal.
2. The error signal being the difference between the reference input signal and the feedback signal obtained from the output.
3. For the system considered as shown in Fig. 2.19.1, the actuating signal is proportional to the error signal; therefore, the system is called proportional control system.
4. Consider a second order system where controller input is error itself and proportional constant is $K = 1$ as shown in Fig. 2.19.1.

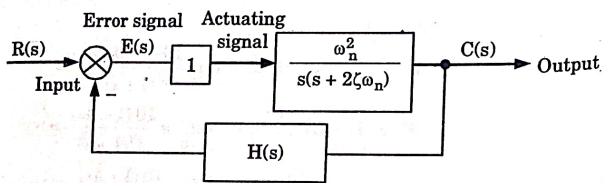


Fig. 2.19.1.

$$G(s)H(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For this system damping ratio is ζ and natural frequency ω_n .

5. And for steady state error

$$K_p = \lim_{s \rightarrow 0} G(s)H(s) = \infty$$

and

$$K_v = \lim_{s \rightarrow 0} s G(s)H(s) = \frac{\omega_n}{2\zeta}$$

B. Advantages :

1. Steady state error is reduced hence the system becomes more stable.
2. Easy to implement.
3. Relative stability is improved.

C. Disadvantages :

1. Due to the presence of these controllers, we get some offsets in the system.
2. Proportional controllers also increase the maximum overshoot of the system.

Que 2.20. Explain derivative and integral control.

Answer

A. Derivative control action :

1. In a controller with derivative control action the output of the controller depends on the rate of change of actuating error signal $e(t)$.
2. A closed loop control system with derivative controller is shown in Fig. 2.20.1.

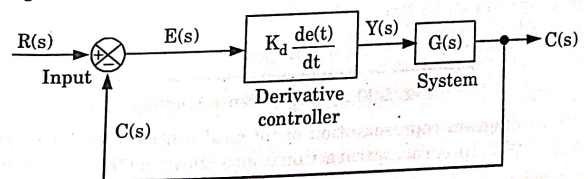


Fig. 2.20.1.

3. Mathematically, the output of a derivative controller is given as :

$$y(t) = K_d \frac{de(t)}{dt} \quad \dots(2.20.1)$$

Here ' K_d ' is known as derivative gain.

4. By taking Laplace transform on both sides of eq. (2.20.1) while assuming zero initial conditions, we get

$$Y(s) = K_d s E(s)$$

5. Putting the value of $E(s) = R(s) - C(s)$ in eq. (2.20.2), we get

$$Y(s) = K_d s [R(s) - C(s)] \quad \dots(2.20.2)$$

Also

$$C(s) = Y(s) G(s)$$

$\therefore$

$$C(s) = G(s) \cdot K_d s [R(s) - C(s)]$$

Thus

$$\frac{C(s)}{R(s)} = \frac{s G(s) K_d}{1 + s K_d G(s)}$$

B. Integral control action :

- In a controller with integral control action, the output of controller is changed at a rate which is proportional to the actuating error signal $e(t)$.
- Mathematically,

$$\frac{dy(t)}{dt} = K_i e(t) \quad \dots(2.20.3)$$

or

$$y(t) = K_i \int_0^t e(t) dt \quad \dots(2.20.4)$$

- Taking Laplace transform on both sides of eq. (2.20.4), we get

$$Y(s) = \frac{K_i E(s)}{s}$$

or

$$\frac{Y(s)}{E(s)} = \frac{K_i}{s} \quad \dots(2.20.5)$$

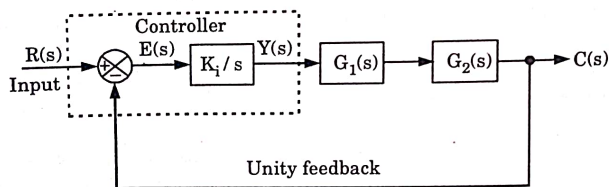


Fig. 2.20.2. Integral control action.

- Block diagram representation of integral control action is shown in Fig. 2.20.2. Integral control action is also known as "Reset control".

Que 2.21. Discuss about proportional derivative (PD) controller.

Answer

- A controller in the forward path, which changes the controller output corresponding to proportional plus derivative of error signal is called PD controller.

- Output of controller = $K e(t) + T_d \frac{de(t)}{dt}$
Taking Laplace transform
 $= K E(s) + s T_d E(s) = E(s) [K + s T_d]$
- Assume, $K = 1$, we can write,

$$G(s) = \frac{(1 + s T_d) \omega_n^2}{s(s + 2\xi \omega_n)}$$

$$\frac{C(s)}{R(s)} = \frac{(1 + s T_d) \omega_n^2}{s^2 + s[2\xi \omega_n + \omega_n^2 T_d] + \omega_n^2}$$

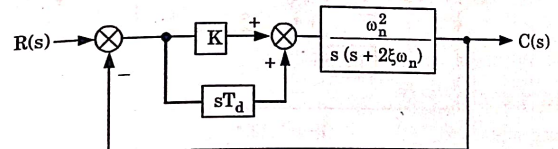


Fig. 2.21.1

- Comparing denominator with standard form,
 $2\xi' \omega_n = 2\xi \omega_n + \omega_n^2 T_d$
 $\therefore \xi' = \xi + \frac{\omega_n T_d}{2}$
- Because of this controller, damping ratio increased by factor $\frac{\omega_n T_d}{2}$.

$$K_p = \lim_{s \rightarrow 0} G(s)H(s) = \infty$$

$$K_v = \lim_{s \rightarrow 0} s G(s)H(s) = \frac{\omega_n}{2\xi}$$

without PD controller,

$$G(s)H(s) = \frac{\omega_n^2}{s(s + 2\xi \omega_n)} \quad [K = 1]$$

$$K_v = \lim_{s \rightarrow 0} s G(s)H(s) = \frac{\omega_n}{2\xi}$$

As there is no change in coefficients, error also will remain same.

Que 2.22. Explain P, PI, PID controllers and also give their advantages.

AKTU 2018-19, Marks 07

Answer

- Proportional control :** Refer Q. 2.19, Page 2-27C, Unit-2.

B. Proportional integral controller (PI) :

1. A controller in the forward path, which changes the controller output corresponding to the proportional plus integral of the error signal is called PI controller.
2. Output of controller = $K e(t) + K_i \int e(t) dt$

Taking Laplace = $K E(s) + \frac{K_i}{s} E(s) = E(s) \left[K + \frac{K_i}{s} \right]$

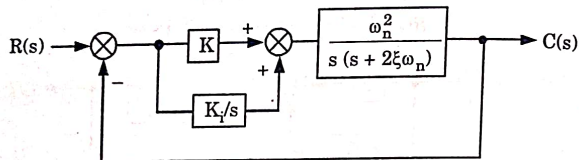


Fig. 2.22.1.

3. Assume, $K = 1$, we can write,

$$G(s) = \frac{\left[1 + \frac{K_i}{s} \right] \omega_n^2}{s(s + 2\xi\omega_n)} = \frac{(K_i + s) \omega_n^2}{s^2(s + 2\xi\omega_n)}$$

i.e., system becomes TYPE 2 in nature.

$$\frac{C(s)}{R(s)} = \frac{(K_i + s) \omega_n^2}{s^3 + 2\xi\omega_n s^2 + s\omega_n^2 + K_i\omega_n^2}$$

i.e., it becomes third order.

4. Now as order increases by one, system relatively becomes less stable as K_i must be designed in such a way that system will remain in stable condition. Second order system is always stable.

while $K_p = \lim_{s \rightarrow 0} G(s)H(s) = \infty, e_{ss} = 0$

$$K_v = \lim_{s \rightarrow 0} s G(s)H(s) = \infty, e_{ss} = 0$$

5. Hence as type is increased by one, error becomes zero for ramp type of input i.e., steady state of system gets improved and system becomes more accurate in nature.

6. Advantages :

- i. Good transient response.
- ii. Helps in stabilizing the controller gain.
- iii. Eliminates the offset error.
- iv. Provides better stability.

C. PID controller (Proportional, integral and derivative controller) :

1. It is a close loop system which has feedback control system and it compares the process variable (feedback variable) with set point and

generates an error signal and according to that it adjusts the output of system.

2. It is the combination of proportional, integral and derivative controller.
3. A PD (Proportional derivative) type of controller improves transient part without affecting the steady state part.
4. While PI (Proportional integral) type of controller improves steady state part without affecting the transient part.
5. In PID, both PI and PD effects are incorporated. Hence both transient as well as steady state part of the response can be improved.
6. This can be realized as shown in the Fig. 2.22.2.

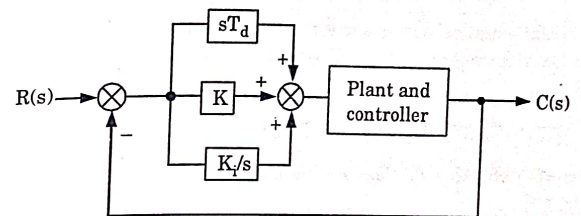


Fig. 2.22.2.

7. The output of PID controller is time domain is,

$$\text{Controller output} = Ke(t) + K_i \int e(t) dt + T_d \frac{de(t)}{dt}$$

8. Taking the Laplace transform, controller output in s-domain is,

$$\text{Controller output} = E(s) \left[K + \frac{K_i}{s} + sT_d \right]$$

9. Advantages :

- i. Improving the stability of the system.
- ii. Reduces the steady-state error.
- iii. Feasible and easy to implement
- iv. Makes the system faster by reducing the time constant

Que 2.23. Compare Proportional (P) control action with Integral (I) control action and prove that by using proportional integral (PI) controller steady state error become zero in a system.

AKTU 2021-22, Marks 10

Answer**A. Comparison :**

S. No.	Proportional control	Integral control
1.	In proportional control the actuating signal for the control action in a control system is proportional to the error signal.	In a controller with integral control action, the output of controller is changed at a rate which is proportional to the actuating error signal $e(t)$.
2.	Mathematically, it is written as $y(t) = K_p e(t)$.	Mathematically, it is written as $y(t) = K_i \int_0^t e(t) dt$

B. Proof : Refer Q. 2.22, Page 2-30A, Unit-2.

Que 2.24. What is Ziegler-Nichols rules for tuning PID controllers ?
How to design a controller using Ziegler Nichols ?

Answer**A. Ziegler-Nichols rules for tuning PID controllers :**

1. Ziegler-Nichols is a widely used tuning method for PID (Proportional-Integral-Derivative) controllers.
2. It provides a set of rules to calculate the controller parameters based on the system's response to a step change, specifically its reaction curve or oscillatory behavior.
3. These rules aim to provide a good balance between stability and fast response.

B. Designing controller using Ziegler Nichols : The Ziegler-Nichols method can be applied in two forms :

1. **Ziegler-Nichols open-loop tuning (Step response method / Reaction curve method) :**
 - i. Apply a step input to the system and measure the delay time L and the time constant T from the response curve.
 - ii. Use these parameters in Ziegler-Nichols formulas to compute the PID gains based on the controller type (P, PI, or PID).
2. **Ziegler-Nichols closed-loop tuning (Ultimate gain method) :**
 - i. Set up a proportional-only controller and gradually increase the gain K_p until the system oscillates with a constant amplitude.
 - ii. Record the ultimate gain K_u and ultimate period P_u .
 - iii. Apply the Ziegler-Nichols tuning rules to calculate the PID parameters based on K_u and P_u .

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain various standard test signals, and also find relation between them.

Ans. Refer Q. 2.1, Unit-2.

Q. 2. Derive the time response of first order system.

Ans. Refer Q. 2.2, Unit-2.

Q. 3. Derive the expressions for second order system for underdamped case and when the input is unit step.

Ans. Refer Q. 2.4, Unit-2.

Q. 4. The open loop transfer function of a unity feedback control system is given by $G(s) = 9/s(s + 3)$. Find the natural frequency of response, damping ratio, damped frequency and time constant.

Ans. Refer Q. 2.6, Unit-2.

Q. 5. Define the following term :

- i. Rise time
- ii. Peak time
- iii. Peak overshoot
- iv. Settling time

Ans. Refer Q. 2.7, Unit-2.

Q. 6. The unity feedback system is characterized by an open loop transfer function is $G(S) = K/s(s + 20)$. Determine the gain K , so that the system will have a damping ratio of 0.6. For this value of K , determine unit step response, time domain specifications: settling time (2% criterion), peak overshoot, rise time, peak time, delay time for a unit-step input.

Ans. Refer Q. 2.9, Unit-2.

Q. 7. Derive the expression for steady state error and explain K_p , K_v and K_a .

Ans. Refer Q. 2.13, Unit-2.



3

UNIT

Stability and Algebraic Criteria

CONTENTS

- Part-1** : Concept of Stability and its 3-2C to 3-5C
Necessary Conditions
- Part-2** : Routh-Hurwitz Criteria and 3-5C to 3-11C
its Limitations
- Part-3** : Root Locus Technique : Salient 3-11C to 3-11C
Features of Root Locus Plot
- Part-4** : Procedure for Plotting Root 3-12C to 3-26C
Locus, Root Contours

3-1 C (EN-Sem-5)

3-2 C (EN-Sem-5)

Stability and Algebraic Criteria

PART-1

Concept of Stability and its Necessary Conditions.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. Define stability. Also write the necessary conditions for stability ?

Answer

A. Stability :

1. A system is stable if its response (*i.e.*, the transfer function) approaches zero as time approaches infinity.
2. In other words, a system is stable if every bounded input produces a bounded output.

B. Types of stability :

1. **Absolute stability** : If a system output is stable for all variations of its parameters, then the system is called absolutely stable system. It gives the information about whether system is stable or unstable.
2. **Relative stability** : The system is said to be relatively more stable or unstable on the basis of settling time. System is said to be relatively more stable if settling time for the system is less than that of the other system. Relative stability gives degree of stability or how close it to instability.

C. Necessary conditions for stability :

1. All the coefficients of characteristics equation must have same sign.
2. There should be no missing term.
3. All poles of transfer function should be in left half of s -plane.
4. The degree of denominator polynomial of transfer function is greater or equal to that of numerator polynomial.

Que 3.2. Explain stability on basis of location of poles and zeros.

Answer

1. The linear system is stable if the rightmost pole(s) is/are on the left-hand half plane (LHHP) on the s -plane.
2. The linear system is marginally stable if the rightmost pole(s) is/are simple order (first-order) on the $j\omega$ axis, including the origin on the s -plane.

- The linear system is unstable if the rightmost pole(s) is/are on the right-hand half plane (RHHP) of the s-plane or if the rightmost pole(s) is/are multiple-order on the $j\omega$ axis on the s-plane.
- Zeros do not affect system Stability.

Que 3.3. Explain the effect of pole location on stability of a system by suitable diagram.

Answer

Effect of pole location on stability :

a. Poles on negative real axis :

- Consider a simple pole at $s = -a$ in Fig. 3.3.1, the corresponding impulse response is given by

$$c(t) = g(t) = L^{-1}[G(s)] = L^{-1}\left[\frac{K}{s+a}\right] = Ke^{-at} \quad \dots(3.3.1)$$

- As the time t increases, the response approaches zero and the system is stable. The response is shown in Fig. 3.3.1(b).

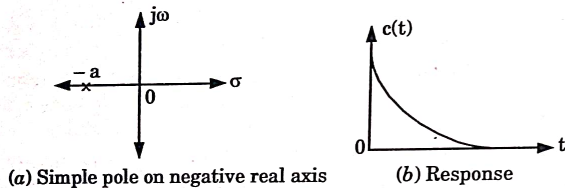


Fig. 3.3.1.

b. Pole on positive real axis :

- Consider a system having simple pole on positive real axis at $s = a$, the corresponding impulse response is given by

$$c(t) = g(t) = L^{-1}\left[\frac{K}{s+a}\right] = Ke^{at} \quad \dots(3.3.2)$$

- The response increases exponentially with time, hence the system is unstable. The simple pole and response are shown in Fig. 3.3.2.

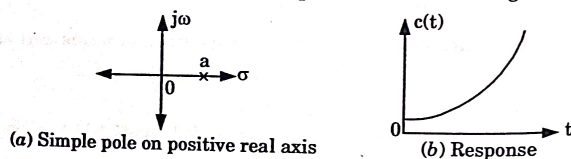


Fig. 3.3.2.

c. Pole at the origin : Consider a pole at origin,

$$g(t) = L^{-1}\left[\frac{K}{s}\right] = K \quad \dots(3.3.3)$$

This is constant value; hence the system is marginally stable. If there are two poles at the origin, the time response would be

$$g(t) = L^{-1}\left[\frac{K}{s^2}\right] = Kt \quad \dots(3.3.4)$$

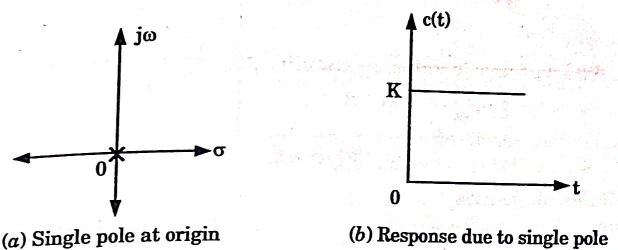


Fig. 3.3.3.

d. Complex pole in the left half of s-plane :

- Let the transfer function has a complex conjugate poles at $s = -\alpha \pm j\omega$. The impulse response due to the complex conjugate poles is given by

$$c(t) = g(t) = L^{-1}\left[\frac{K}{s + \alpha - j\omega} + \frac{K}{s + \alpha + j\omega}\right] = L^{-1}\left[\frac{2K(s + \alpha)}{(s + \alpha)^2 + \omega^2}\right] = 2Ke^{-\alpha t} \cos \omega t \quad \dots(3.3.5)$$

- When t increases $g(t)$ approaches zero and the system is stable. The complex poles and corresponding time response is shown in Fig. 3.3.4(a) and Fig. 3.3.4(b) respectively.

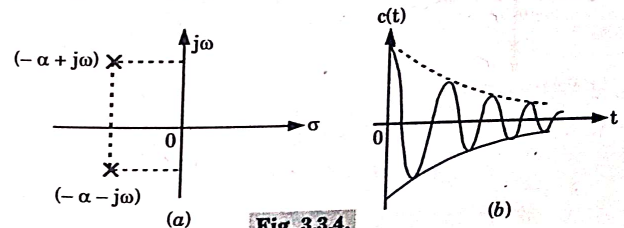


Fig. 3.3.4.

e. Complex poles in the right half of s-plane :

- Suppose the system has complex conjugate poles at $s = \alpha \pm j\omega$. The time response is given by

$$c(t) = g(t) = L^{-1}\left[\frac{K}{s - \alpha - j\omega} + \frac{A}{s - \alpha + j\omega}\right]$$

$$= L^{-1} \left[\frac{2A(s-\alpha)}{(s-\alpha)^2 + \omega^2} \right] = 2Ae^{\alpha t} \cos \omega t \quad \dots(3.3.6)$$

2. Hence, the response increases exponentially sinusoid with time and therefore the response is unstable. The poles and time response is shown in Fig. 3.3.5(a) and Fig. 3.3.5(b) respectively.

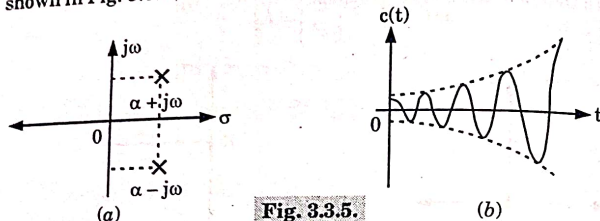


Fig. 3.3.5.

f. Poles on $j\omega$ -axis :

1. If the system having the complex poles on $j\omega$ -axis the corresponding time response would be

$$g(t) = L^{-1} \left[\frac{A}{s + j\omega} + \frac{A}{s - j\omega} \right]$$

$$= L^{-1} \left[\frac{2As}{s^2 + \omega^2} \right] = 2A \cos \omega t \quad \dots(3.3.7)$$

2. The response is marginally stable. Equation (3.3.7) shows the sustained oscillations of constant amplitude. This situation will also be considered unstable.

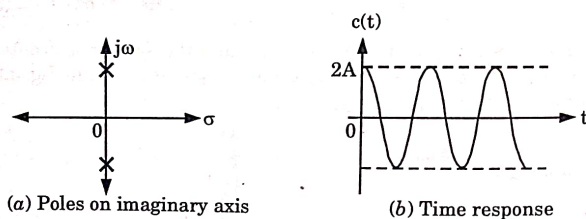


Fig. 3.3.6.

PART-2

Routh-Hurwitz Criteria and its Limitations.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.4. What is Routh-Hurwitz stability criterion? Explain briefly.

OR

What are the limitations and applications of Routh-Hurwitz criterion?

Answer

- A. The Routh-Hurwitz stability criterion is a mathematical test that is a necessary and sufficient condition for the stability of system.
- B. **Routh's criterion :**
1. The necessary and sufficient condition for the system to be stable is that all the terms in the first column of Routh's array (i.e., $a_0, a_1, b_1, c_1, d_1, \dots, a_n$) must have same sign.
 2. There should not be any sign change in the first column of Routh's array.
 3. If there is any sign change then
 - i. System is unstable.
 - ii. Number of sign changes is equal to the number of poles lying in the right half of the s -plane.
- C. **Method of forming a Routh array :**

s^n	a_0	a_2	a_4	a_6	
s^{n-1}	a_1	a_3	a_5	a_7	
s^{n-2}	b_1	b_2	b_3		
s^{n-3}	c_1	c_2	c_3		
$\vdots$	$\vdots$	$\vdots$	$\vdots$		
s^0	a_n				

1. Coefficients for first two rows are written directly from characteristics equation,

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

2. From 2nd and 3rd row, 4th row can be obtained as

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

3. This process is to be continued till the coefficient for s_0 is obtained which will be a_n . From this array stability of a system can be predicated.

D. Special cases :

- When the 1st term in a row is zero, but all other terms are non-zero then substitute a small positive number ϵ for zero & proceed to evaluate the rest of the elements. When the 1st column term is zero, it means that there is an imaginary root.
 - All zero row :** In this case, write auxiliary equation from preceding row, differentiate this equation & substitute all zero row by the coefficient obtained by differentiating the auxiliary equation. This case occurs when the roots are in pairs. The system is said to be limitedly stable.
- E. Application :** Routh's criterion can be applied to determine range of certain parameters of a system to ensure stability. It is usually to find the range of the open loop gain K for closed loop stability.

F. Limitations :

- It is valid only if the characteristics equation is algebraic.
- If any coefficient of the characteristics equation is complex or contain power of e then this criterion cannot be applied.
- It gives information about how many roots are lying in the RHS of the s -plane but values of the roots are not available. Also it cannot distinguish between real and complex roots.

Que 3.5. Explain stability on basis of location of poles and zeros.

For a unity feedback system $G(s) = k/s(s+1)(1+2s)(1+3s)$.

Determine range of k for stability, value of k for frequency of sustain oscillation.

AKTU 2019-20, Marks 07

Answer

A. Stability on basis of location : Refer Q. 3.2, Page 3-2C, Unit-3.

B. Numerical :

Given : $G(s) = \frac{k}{s(s+1)(1+2s)(1+3s)}$, $H(s) = 1$

To Find : Range of k , Value of k for frequency of sustain oscillation.

1. Characteristics equation,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k}{s(s+1)(1+2s)(1+3s)} = 0$$

$$s(s+1)(1+2s)(1+3s) + k = 0$$

$$6s^4 + 11s^3 + 6s^2 + s + k = 0$$

s^4	6	6	k
s^3	11	1	0
s^2	$\frac{60}{11}$	k	0
s^1	$\frac{60}{11} - 11k$	0	
s^0	$\frac{60}{11}$		
	k		

- For system to be stable there should not be sign change in the first column.
 $\therefore k > 0$ for s^0 and $60 - 121k > 0$
 $60 > 121k$
 $0.49 > k, k < 0.49$
- So the range of k is $0 < k < 0.49$
 $k_{\text{mar}} = 0.49$
- To find frequency, find out roots of auxiliary equation at marginal value of k

$$A(s) = \frac{60}{11}s^2 + k = 0$$

$$60s^2 + 11k = 0$$

$$60s^2 + 11 \times 0.49 = 0$$

$$60s^2 + 5.39 = 0$$

$$60s^2 = -5.39$$

$$s^2 = -5.39/60 = 0.089$$

$$s = \sqrt{-0.089} = \pm j0.298$$

- Comparing with $s = \pm j\omega$
 So, ω = Frequency of oscillations = 0.298 rad/sec.

Que 3.6. Determine the stability of a closed loop control system whose characteristic equation is $s^5 + s^4 + 2s^3 + 2s^2 + 11s + 10 = 0$

Answer

Given : Characteristic equation is $s^5 + s^4 + 2s^3 + 2s^2 + 11s + 10 = 0$
 To Find : Stability.

1. Routh array is :

s^5	1	2	11
s^4	1	2	10
s^3	0	1	
s^2			
s^1			
s^0			

2. Putting ϵ (very small positive value) in place of 0 in the first column
Routh array

s^5	1	2	11
s^4	1	2	10
s^3	ϵ (+ve)	1	0
s^2	$\frac{2\epsilon - 1}{\epsilon}$ (-ve)	10	
s^1	$\frac{\left(\frac{2\epsilon - 1}{\epsilon}\right) - 10\epsilon}{\left(\frac{2\epsilon - 1}{\epsilon}\right)}$ (+ve)	0	
s^0	10		

3. Since there is two sign change in the first column of Routh array, therefore system is unstable.

Que 3.7. Determine the Routh's stability of given characteristics

equation $s^4 + 18s^3 + 18s^2 + 16s + 5 = 0$.

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Answer

Given: $G(s) = s^4 + 18s^3 + 18s^2 + 16s + 5 = 0$

To Find: Stability.

1. Routh array,

s^4	1	18	5
s^3	8	16	0
s^2	16	5	0
s^1	13.5	0	
s^0	5		

2. Here no sign changes hence given system is stable.

Que 3.8. Explain the effect of pole location on stability of a system by suitable diagram. Determine range of K and frequency of sustained oscillations for a given unity feedback system.

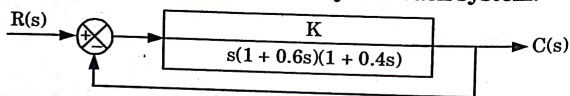


Fig. 3.8.1.

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Answer

- A. Effect of pole location on stability: Refer Q. 3.3, Page 3-3C, Unit-3.
B. Numerical:

Given: $G(s) = \frac{K}{s(1+0.6s)(1+0.4s)}$, $H(s) = 1$

To Find: Range of k , frequency of sustained oscillations.

1. The characteristic equation

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(1+0.6s)(1+0.4s)} \times 1 = 0$$

$$s(1+0.6s)(1+0.4s) + K = 0$$

$$0.24s^3 + s^2 + s + K = 0$$

2. The Routh array

s^3	0.24	1
s^2	1	K
s^1	$1 - 0.24K$	0
s^0	K	

3. For system to be stable

From row of s^0

Thus row of s^1

$$K > 0$$

$$1 - 0.24K > 0$$

$$1 > 0.24K$$

$$K < 4.16$$

$$0 < K < 4.16$$

This is the range of K

4. For frequency of sustained oscillations

Auxiliary equation

$$A(s) = s^2 + K = 0$$

$$s^2 = -K \quad \begin{cases} 1 - 0.24K = 0 \\ K = 4.16 \end{cases}$$

At $K = 4.16$,

$$s^2 = -4.16$$

$$s = \pm j 2.03$$

5. The frequency of sustained oscillations will be 2.03 rad/sec.

Que 3.9. What is the necessary condition for stability? Explain limitation of Routh's stability method, construct Routh array and determine the stability of the system whose characteristics equation, $s^5 + 2s^4 + s^3 + 2s^2 + s + 4 = 0$.

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Answer

- i. Necessary condition: Refer Q. 3.1, Page 3-2C, Unit-3.

- ii. **Limitations :** Refer Q. 3.4, Page 3-6C, Unit-3.
 iii. **Numerical :** The procedure is same as Refer Q. 3.6, Page 3-8C, Unit-3.
Ans.
 The given system is unstable.

PART-3

Root Locus Technique : Salient Features of Root Locus Plot.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.10. What is root locus ? Also give the salient features of root locus.

Answer**A. Root Locus :**

1. Root locus is the plot of loci of the root of the complementary equation when one or more parameter of open-loop transfer function is varied; mostly the only one variable available is the gain K . The negative K has no significance. Hence vary K from 0 to ∞ , the plot obtained is called root locus.
2. It gives the complete dynamic response of system. It provides the measure of sensitivity of roots of variation in the system.

B. Features :

1. The root locus starts ($K = 0$) from the open loop poles and terminates ($K = \infty$) on either finite open loop zeros or infinity.
2. The number of separate branches of the root locus equals either the number of open loop poles or number of open loop zeros whichever is greater.
3. A section of root locus lies on the real axis if the total number of open loop poles and zeros to the right of the section is odd.
4. The root locus is symmetrical with respect to the real axis.
5. At some points on the root locus between two open loop poles the root locus breaks away.
6. For higher values of K the root locus can be approximated by asymptotic lines and these asymptotic lines intersect at a point on the real axis.
7. If the root locus intersects the imaginary axis then the point of intersection is conjugate.

PART-4

Procedure for Plotting Root Locus, Root Contours.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.11. Explain step by step procedure for plotting root locus. Also explain root contour.

Answer**Rules for construction of root locus :**

1. The root locus is symmetrical about the real axis.
2. Each branch of the root locus originates from an open-loop pole where $K = 0$ and terminates on an open loop zero or on infinity where $K = \infty$.
3. The number of branches of the root locus terminating on infinity is equal to $P - Z$.
4. A point on the real axis lies on the locus, if number of open loop poles and zeros on the real axis to the right of this point is odd.
5. **Angles of asymptotes :** The $(P - Z)$ root locus branches that approach to infinity are along straight line is called asymptote. Asymptotes making angles with the real axis are given by

$$\theta_q = \frac{(2q+1)180^\circ}{P-Z}; \text{ where } q = 0, 1, 2, \dots, (P-Z-1)$$

6. **Centroid :** The point of intersection of the asymptotes with the real axis called centroid is at $s = \sigma$, where

$$\sigma = \frac{\sum \text{Real part of poles} - \sum \text{Real part of zeros}}{\text{Number of poles} - \text{Number of zeros}}$$

7. **Break points :** The break points (breakaway and breakin points) of the root locus are determined from the roots of the equation $\frac{dK}{ds} = 0$.

The r branches of the root locus which meet at a point break away at an angle of $\pm \frac{180^\circ}{r}$.

8. **Angle of departure**: The angle of departure from an open loop pole is given by

$$\theta_d = 180^\circ + \phi;$$

where ϕ is the net angle contribution at the zero of all other open-loop poles and zeros.

9. **Angle of arrival**: The angle of arrival at open loop zero is given by

$$\theta_a = 180^\circ - \phi,$$

where ϕ is the net angle contribution of all other open loop poles and zeros.

10. The point of intersection of the root locus branches with the imaginary axis and the critical value of K can be determined by use of the Routh's criterion.

11. **Gain**: The open-loop gain K in pole-zero form at any point s_0 on the root locus is given by

$$K = \frac{\text{Product of phasor lengths from } s_0 \text{ to open-loop poles}}{\text{Product of phasor lengths from } s_0 \text{ to open-loop zeros}}$$

Root contour:

- The root locus technique for the study of stability of closed loop system from open loop transfer function can be extended by varying parameter other than K in system from 0 to ∞ .
- Consider a system shown in the Fig. 3.11.1.

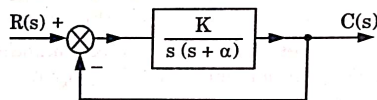


Fig. 3.11.1.

- The open loop transfer function is $G(s)H(s) = \frac{K}{s(s+\alpha)}$
- The locus of roots of characteristic equation obtained by varying parameter of the system other than K from 0 to ∞ is called root contour of the system.
- The parameters like α , K are to be varied simultaneously, while sketching the root contours.

Que 3.12: Sketch the root locus for the open loop transfer function of a unity feedback control system given below and determine value

of K at $\xi = 0.5$, $G(s)H(s) = \frac{K}{s(s+1)(s+3)}$.

Answer

Given: $G(s)H(s) = \frac{K}{s(s+1)(s+3)}$, $H(s) = 1$, $\xi = 0.5$

To Sketch: Root locus and value of K at $\xi = 0.5$.

A.

- The open-loop poles are at $s = 0, s = -1, s = -3$.
- There is no open loop zeros.
- Number of poles, $P = 3$
- Number of zeros, $Z = 0$
 $P - Z = 3 - 0 = 3$ i.e., three branches of root locus end at infinity.
- Angle of asymptotes:**

$$\theta_q = \frac{(2q+1)180^\circ}{P-Z} \quad \text{where } q = 0, 1, 2, \dots (P-Z-1)$$

$$\theta_0 = \frac{(2 \times 0 + 1) \times 180^\circ}{(3-0)} = 60^\circ$$

$$\theta_1 = \frac{(2 \times 1 + 1) \times 180^\circ}{(3-0)} = 180^\circ$$

$$\theta_2 = \frac{(2 \times 2 + 1) \times 180^\circ}{(3-0)} = 300^\circ$$

5. **Centroid of asymptotes:**

$$\sigma = \frac{\sum \text{Real parts of poles} - \sum \text{Real parts of zeros}}{P-Z}$$

$$= \frac{(0 - 1 - 3) - (0)}{3 - 0} = -1.33.$$

6. **Breakaway points:** Between open-loop poles $s = 0$ and $s = -1$, there exists a breakaway point.

The characteristic equation is

$$s(s+1)(s+3) + K = 0$$

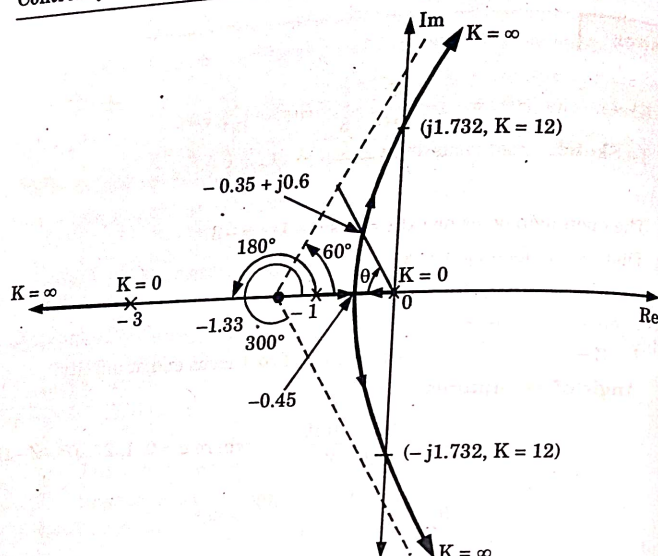
$$K = -(s^3 + 4s^2 + 3s)$$

$$\therefore \frac{dK}{ds} = -(3s^2 + 8s + 3) = 0$$

$$\therefore 3s^2 + 8s + 3 = 0$$

$$s = -1.33 \pm 0.88 = -0.42 \text{ and } -2.21$$

As the breakaway point has to lie between $s = 0$ and $s = -1$, the valid breakaway point is $s = -0.42$.

Fig. 3.12.1. Root locus for $G(s) = K/s(s+1)(s+3)$.7. Intersection with $j\omega$ axis:Characteristic equation, $1 + G(s)H(s) = 0$

$$s(s+1)(s+3) + K = 0$$

$$s^3 + 4s^2 + 3s + K = 0$$

Routh array:

s^3	1	3
s^2	4	K
s^1	$(12-K)/4$	
s^0	K	

The value of K at imaginary axis:

$$\frac{(12-K)}{4} = 0 \quad \therefore K = 12$$

Auxiliary equation,

$$4s^2 + 12 = 0$$

$$s = \pm j 1.73$$

B.

1. Since
and

$$\xi = 0.5$$

$$\cos \theta = \xi$$

$$\cos \theta = 0.5$$

$$\theta = 60^\circ$$

2. From the origin a line at angle $\theta = 60^\circ$ is drawn as shown in Fig. 3.12.1, which intersects the root locus plot at $s = (-0.35 + j0.6)$. As the point $s = (-0.35 + j0.6)$ lies on the root locus the following equation is satisfied

$$|G(s)H(s)| = 1 \quad \text{or} \quad \left| \frac{K}{s(s+1)(s+3)} \right| = 1$$

$$s = (-0.35 + j0.6), \text{ we get}$$

3. Put

$$\left| \frac{K}{(-0.35 + j0.6)[(-0.35 + j0.6) + 1][(-0.35 + j0.6) + 3]} \right| = 1$$

$$\therefore \frac{K}{1.66} = 1 \quad \text{or} \quad K = 1.66$$

Que 3.13. Sketch the root locus of the system, whose transfer

function is $G(s) = K(s+15)/s(s+1)(s+5)$. **AKTU 2020-21, Marks 10**

Answer

Given: $G(s) = \frac{K(s+15)}{s(s+1)(s+5)}$

To Draw: Root locus.

1. Poles i.e., $s = 0, s = -1, s = -5$
2. Zeros, $s = -15$
3. Number of poles, $P = 3$
Number of zeros, $Z = 1$
4. Number of root locus branches = $P - Z = 3 - 1 = 2$ i.e., two branches of root locus end at infinity.
5. Angle of asymptote:

$$\theta_q = \frac{(2q+1)180}{P-Z}$$

where,

$$q = 0, 1, 2, \dots, (P-Z-1)$$

$$\theta_0 = \frac{(2 \times 0 + 1) \times 180}{2} = 90^\circ$$

$$\theta_1 = \frac{(2 \times 1 + 1) \times 180}{2} = 270^\circ$$

6. Centroid of asymptotes:

$$= \frac{\Sigma(\text{Real parts of poles}) - \Sigma(\text{Real parts of zeros})}{P-Z}$$

$$= \frac{(0 - 1 - 5) - (-15)}{2} = \frac{-6 + 15}{2} = 4.5$$

7. Intersection points with imaginary axis :

The characteristic equation is,
 $1 + G(s)H(s) = 0$

$$1 + \frac{K(s+15)}{s(s+1)(s+5)} = 0$$

$$s(s+1)(s+5) + K(s+15) = 0$$

$$s^3 + 6s^2 + s[K+5] + 15K = 0$$

The Routh array :

s^3	1	$K+5$
s^2	6	$15K$
s^1	$\frac{6(K+5) - 15K}{6}$	0
s^0	$15K$	0

$$\frac{6(K+5) - 15K}{6} = 0$$

$$\frac{30 - 9K}{6} = 0$$

$$K = \frac{30}{9} = 3.33$$

Auxiliary equation

$$6s^2 + 15 \times 3.33 = 0, s^2 = \frac{-49.95}{6}$$

$$s = \pm j2.88$$

8. Breakaway point :

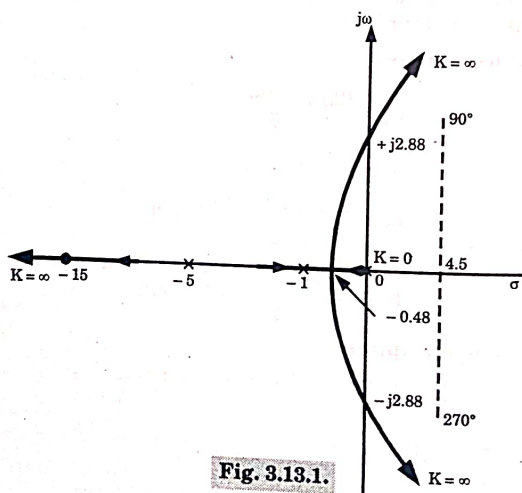


Fig. 3.13.1.

The characteristics equation is,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+15)}{s(s+1)(s+5)} = 0$$

$$(s^3 + 6s^2 + 5s) + K(s+15) = 0$$

$$K = -\frac{[s^3 + 6s^2 + 5s]}{s+15}$$

$$\frac{dK}{ds} = 0$$

$$\frac{d}{ds} \left(\frac{-s^3 - 6s^2 - 5s}{s+15} \right) = 0$$

$$\frac{(s+15)(-3s^2 - 12s - 5) - (-s^3 - 6s^2 - 5s) \times 1}{(s+15)^2} = 0$$

$$2s^3 + 51s^2 + 180s + 75 = 0$$

$$s = -21.37, -0.48, -3.64$$

Que 3.14. A unity feedback system has an open loop transfer function.

Draw the root locus for the system.

$$G(s)H(s) = k \frac{(s+2)}{(s+3)(s^2+2s+2)}$$

AKTU 2019-20, Marks 07

Answer

$$\text{Given : } G(s)H(s) = \frac{k(s+2)}{(s+3)(s^2+2s+2)}$$

To Draw : Root locus.

Step 1 : Poles, $P_1 = -3, P_2 = -1+j, P_3 = -1-j$ Zeros, $Z = -2$

No of poles = 3, No of zeros = 1

Step 2 : No of asymptotes = $P - Z = 3 - 1 = 2$

$$\text{Angle of asymptotes } \theta_1 = \frac{(2k+1) \times 180^\circ}{P-Z} = \frac{(2 \times 0 + 1) \times 180^\circ}{2} = 90^\circ$$

$$\theta_2 = \frac{(2 \times 1 + 1) \times 180^\circ}{2} = 270^\circ$$

Step 3 : Centroid

$$\sigma = \frac{\sum R.P \text{ of poles} - \sum R.P \text{ of zeros}}{P-Z}$$

$$= \frac{(-3-1-1)-(-2)}{2} = -1.5$$

Step 4: Breakaway point: Characteristics equation,
 $1 + G(s)H(s) = 0$

$$1 + \frac{k(s+2)}{(s+3)(s^2+2s+2)} = 0$$

$$(s+3)(s^2+2s+2) + k(s+2) = 0$$

$$s^3 + 5s^2 + 8s + 6 + k(s+2) = 0$$

$$k = \frac{-s^3 - 5s^2 - 8s - 6}{s+2}$$

$$dk/ds = 0$$

$$\frac{d}{ds} \left[\frac{-s^3 - 5s^2 - 8s - 6}{s+2} \right] = 0$$

$$(s+2)[-3s^2 - 10s - 8] - [-s^3 - 5s^2 - 8s - 6] \times 1 = 0$$

$$-2s^3 - 11s^2 - 20s - 10 = 0$$

$$2s^3 + 11s^2 + 20s + 10 = 0$$

$$s = -0.80, -2.34 + 0.84j, -2.34 - 0.84j$$

That breakaway point is not valid

Step 5: Intersection point: Characteristics equation,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k(s+2)}{(s+3)(s^2+2s+2)} = 0$$

$$s^3 + 5s^2 + 8s + 6 + ks + 2k = 0$$

Apply Routh's criteria,

s^3	1	$k+8$
s^2	5	$6+2k$
s^1	$\frac{34+3k}{5}$	0
s^0	$6+2k$	0

$$34 + 3k/5 = 0, \quad 34 + 3k = 0$$

$$3k = -34$$

$$k = -34/3 = -11.33$$

$$\text{Auxiliary equation } 5s^2 + 6 + 2k = 0$$

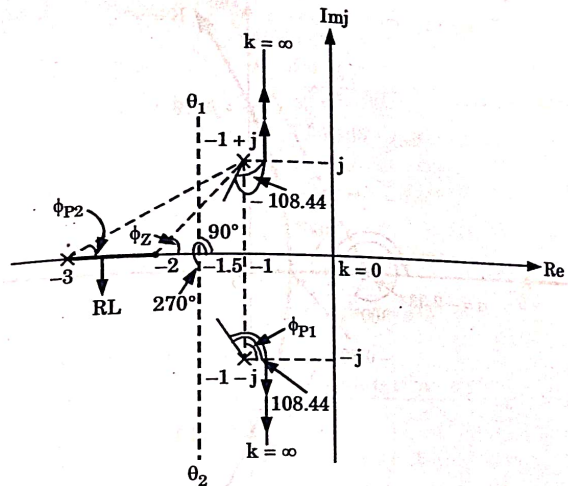


Fig. 3.14.1

$$5s^2 + 6 + 2 \times -11.33 = 0$$

$$s^2 = 3.33 = s = \pm 1.82$$

That intersection point is not valid.

Step-6: Angle of departure:

$$\phi_d = 180^\circ - [\phi_P - \phi_Z]$$

$$\phi_{P1} = 90^\circ, \phi_{P2} = 26.56^\circ$$

$$\phi_Z = 45^\circ$$

$$\phi_d = 180^\circ - [90^\circ + 26.56^\circ - 45^\circ] = 108.44^\circ$$

Que 3.15. Sketch the root locus of the system whose open loop transfer function is $G(S) = K/s(s+2)(s+5)$. Find the value of K so that system is marginal stable find out damped frequency of oscillation, also find K when the damping ratio of the closed loop system is 0.5.

AKTU 2021-22, Marks 10

Answer

The procedure is same as Q. 3.12, Page 3-13C, Unit-3.

1. Root locus :

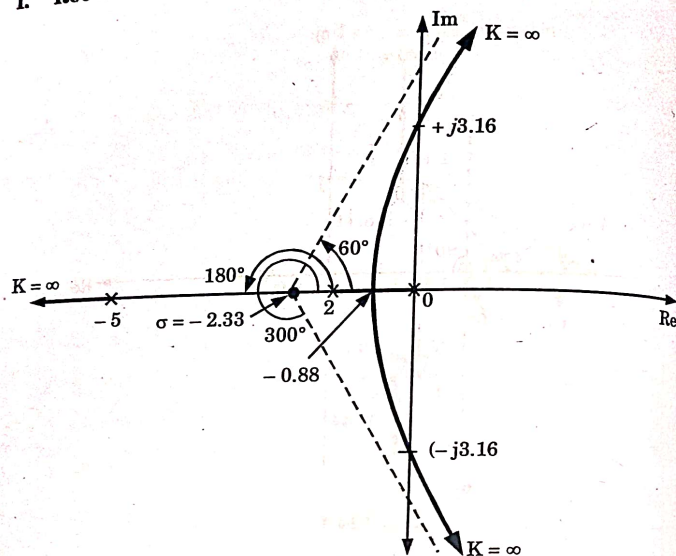


Fig. 3.15.1.

- The value of K for marginally stable system, $K = 70$
- The damped frequency of oscillation, $\omega_n = 1.44$ rad/sec
- The value of K when the damping ratio, $\xi = 0.5$

$$K = 10.75$$

Que 3.16. Sketch the root locus for the closed loop control system

with $G(s) = \frac{K}{s(s+1)(s^2+4s+5)}$.

Answer

Given : $G(s) = \frac{K}{s(s+1)(s^2+4s+5)}$

To Draw : Root locus.

- Poles i.e., $s = 0, s = -1, s = -2 + j$ and $s = -2 - j$
- There is no open loop zero.
- Number of poles, $P = 4$
Number of zeros, $Z = 0$

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Stability and Algebraic Criteria

$P - Z = 4 - 0 = 4$ i.e., four branches of root locus terminates at infinity.

4. Angle of asymptotes :

$$\theta_q = \frac{(2q+1)}{P-Z} \times 180^\circ$$

where, $q = 0, 1, 2, \dots$ upto $(P-Z-1)$

$$\theta_0 = \frac{2 \times 0 + 1}{4 - 0} \times 180^\circ = 45^\circ$$

$$\theta_1 = \frac{2 \times 1 + 1}{4 - 0} \times 180^\circ = 135^\circ$$

$$\theta_2 = \frac{2 \times 2 + 1}{4 - 0} \times 180^\circ = 225^\circ$$

$$\theta_3 = \frac{2 \times 3 + 1}{4 - 0} \times 180^\circ = 315^\circ$$

5. Centroid of asymptotes :

$$\sigma = \frac{\sum \text{Real parts of poles} - \sum \text{Real parts of zeros}}{P - Z}$$

$$= \frac{(0 - 1 - 2 + j - 2 - j) - (0)}{4 - 0} = \frac{-5}{4} = -1.25$$

6. Breakaway points : The characteristic equation is $1 + G(s)H(s) = 0$

$$s(s+1)(s^2+4s+5) + K = 0$$

$$s^4 + 5s^3 + 9s^2 + 5s + K = 0$$

$$\therefore \frac{dK}{ds} = 4s^3 + (5 \times 3s^2) + (9 \times 2s) + 5 \times 1 = 0$$

$$4s^3 + 15s^2 + 18s + 5 = 0$$

$$(s + 0.4)(4s^2 + 13.4s + 12.5) = 0$$

$$(s + 0.4)\{(s + 1.675 - j 0.565)(s + 1.675 + j 0.565)\} = 0$$

Therefore three breakaway points are obtained

$s = -0.4$ on real axis and

$s = -1.7 + j 0.6$ and $s = -1.7 - j 0.6$

7. Intersection points with imaginary axis :

The characteristic equation is $s^4 + 5s^3 + 9s^2 + 5s + K = 0$

The Routh array :

s^4	1	9	K
s^3	5	5	0
s^2	8	K	
s^1	$5 - \frac{5K}{8}$	0	
s^0	K		

Value of K at imaginary axis :

$$5 - \frac{5K}{8} = 0; K = 8$$

Solving auxiliary equation formed from the s^2 terms in Routh array, therefore

$$8s^2 + K = 0$$

$$8s^2 + 8 = 0$$

$$s = \pm j$$

8. Angle of departure from complex pole :

$$\phi_d = 180^\circ - (\phi_P - \phi_Z)$$

$$\phi_{P1} = 180^\circ - \tan^{-1}(1/2) = 153.43^\circ$$

$$\phi_{P2} = 180^\circ - \tan^{-1}(1/1) = 135^\circ$$

$$\phi_{P3} = 90^\circ$$

$$\phi_{(-2+j)} = 180^\circ - (\phi_{P1} + \phi_{P2} + \phi_{P3})$$

$$\phi_{(-2+j)} = 180^\circ - (153.43^\circ + 135^\circ + 90^\circ) = -198.43^\circ$$

and

$$\phi_{(-2-j)} = +198.43^\circ$$

9. Root locus :

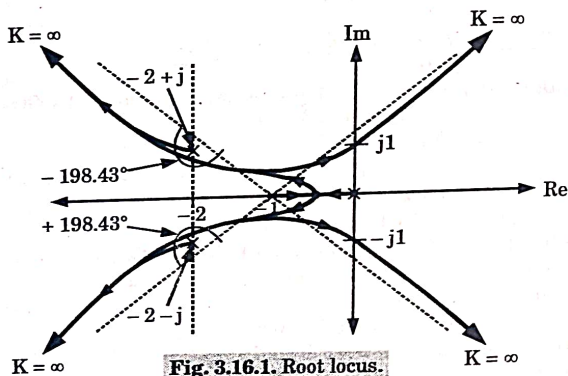


Fig. 3.16.1. Root locus.

Que 3.17. For a unity feedback system the open loop transfer function is given by

$$G(s) = K/s(s+2)(s^2+6s+25)$$

- Sketch the root locus.
- At what value of K the system become unstable.
- At this point of instability determine the frequency of oscillation of the system.

AKTU 2018-19, Marks 07

Answer

The procedure is same as Q. 3.16, Page 3-21C, Unit-3.

3-24 C (EN-Sem-5)

i. Root locus :

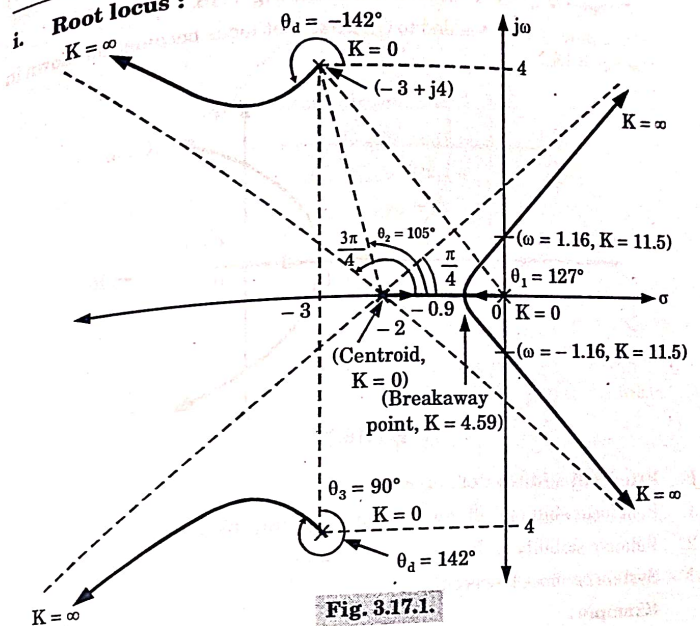


Fig. 3.17.1.

ii. $K = 11.55$

iii. $\omega = 1.16$ rad/sec

Que 3.18. Explain the effect of addition of pole and zero on root locus and time domain specifications. AKTU 2021-22, Marks 10

Answer

A. Effects of addition of poles :

- Root locus shifts towards imaginary axis.
- System stability relatively decreases.
- System becomes more oscillatory in nature.

Example :

- Consider, $G(s)H(s) = K/s(s+2)$

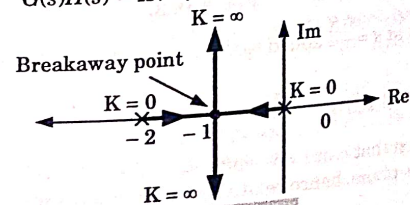


Fig. 3.18.1.

Corresponding root locus is shown in the Fig. 3.18.1.

2. Now if pole at $s = -4$ added to $G(s)H(s)$ root locus becomes as shown in the Fig. 3.18.2.

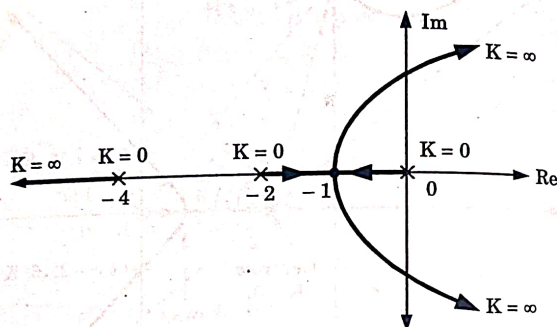


Fig. 3.18.2.

B. Effects of addition of zeros :

1. Root locus shifts to left, away from imaginary axis.
2. Relative stability of the system increases.
3. System becomes less oscillatory.

Example :

1. Root loci for $G(s)H(s) = \frac{K}{s(s+2)}$ is shown in the Fig. 3.18.3.

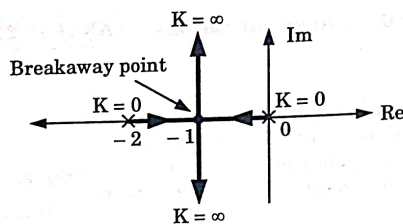


Fig. 3.18.3.

2. Now if zero at $s = -4$ added to $G(s)H(s)$ root locus becomes as shown in Fig. 3.18.4.

$$G(s)H(s) = \frac{K(s+4)}{s(s+2)}$$

3. It can be seen that root locus shift towards left. So as roots move towards left half of s-plane, hence relative stability increases.

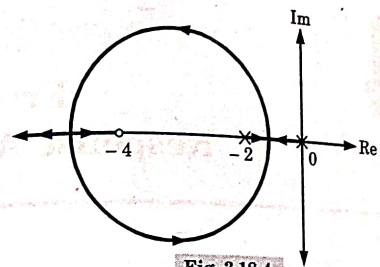


Fig. 3.18.4.

C. Time domain specifications : Refer Q. 2.7, Page 2-11C, Unit-2.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

- Q. 1. Define stability. Also write the necessary conditions for stability ?

Ans. Refer Q. 3.1, Unit-3.

- Q. 2. Explain stability on basis of location of poles and zeros. For a unity feedback system $G(s) = k/s(s+1)(1+2s)(1+3s)$. Determine range of k for stability, value of k for frequency of sustain oscillation.

Ans. Refer Q. 3.5, Unit-3.

- Q. 3. What is the necessary condition for stability ? Explain limitation of Routh's stability method, construct Routh array and determine the stability of the system whose characteristics equation, $s^5 + 2s^4 + s^3 + 2s^2 + s + 4 = 0$.

Ans. Refer Q. 3.9, Unit-3.

- Q. 4. Sketch the root locus of the system, whose transfer function is $G(s) = K(s+15)/s(s+1)(s+5)$.

Ans. Refer Q. 3.13, Unit-3.

- Q. 5. Explain the effect of addition of pole and zero on root locus and time domain specifications.

Ans. Refer Q. 3.18, Unit-3.



4

UNIT

Frequency Response Analysis

CONTENTS

- Part-1** : Frequency Response Analysis 4-2A to 4-3A
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4-1 C (EN-Sem-5)

4-2 C (EN-Sem-5)

Frequency Response Analysis

PART-1

Frequency Response Analysis From Transfer Function Model.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. What do you understand by frequency response? What are its various methods? Discuss their advantages, disadvantages and limitations.

Answer

A. Frequency response :

1. The magnitude and phase relationship between the sinusoidal input and the steady state output of a system is termed as the frequency response.
2. Linear system with sinusoidal input
$$r(t) = A \sin \omega t$$
3. Steady state output of
$$c(t) = B \sin (\omega t + \phi)$$

B. Various methods of frequency response :

1. Polar plots
2. Inverse polar plots
3. Nyquist stability criterion
4. Bode plot

C. Advantages of frequency domain approach :

1. Without the knowledge of the transfer function, the frequency response of stable open loop system can be obtained experimentally.
2. These methods are easy to use for design of control and for finding absolute as well as relative stability of the system.
3. Even if the system has moderate non-linearity it can be depicted by an approximate transfer function.
4. There is a close relation between frequency response of a system and its step response.

D. Disadvantages of frequency response :

1. For systems with very large time constants, the frequency test is cumbersome to perform as the time required for the output to reach the steady-state for each frequency of the test signal is excessively long.
2. The frequency response test cannot be performed on non-interruptible systems.

E. Limitations of frequency response methods :

1. These methods can be applied only to linear systems.
2. For an existing system, obtaining frequency response is possible only if the time constants are up to few minutes.
3. Obtaining frequency response practically is fairly time consuming.

PART-2*Construction of Polar and Inverse Polar Plots.***Questions-Answers****Long Answer Type and Medium Answer Type Questions**

Que 4.2. Explain Polar plot. Discuss phase margin and gain margin on polar plot.

Answer**A. Polar plot :**

1. Polar plot of a transfer function $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar coordinates as ω is varied from zero to infinity. Therefore it is the locus of vectors $|G(j\omega)| \angle G(j\omega)$ as ω is varied from zero to infinity.
2. In frequency response we have

$$M = |G(j\omega) H(j\omega)| = \text{Magnitude}$$

$$\phi = \angle G(j\omega) H(j\omega) = \text{Phase}$$
3. Values of M and ϕ are obtained by varying the input frequency ω from 0 to ∞ . Table 4.2.1 is used to draw polar plot.

Table 4.2.1.

ω	$M = G(j\omega) H(j\omega) $	$\phi = \angle G(j\omega) H(j\omega)$
0	M_0	ϕ_0
ω_1	M_1	ϕ_1
ω_2	M_2	ϕ_2
$\vdots$	$\vdots$	$\vdots$
∞	M_∞	ϕ_∞

4. **Advantage :** It shows the frequency response characteristics of a system over the entire frequency range in a single plot.
5. **Disadvantage :** It does not indicate the contribution of each individual factor of the open loop transfer function.

B. Procedure to sketch the polar plot :

1. Determine $G(s)$.
2. Put $s = j\omega$ in $G(s)$.
3. Evaluate $|G(j\omega)|$ at $\omega = 0$ and $\omega = \infty$.
4. Evaluate phase angle at $\omega = 0$ and $\omega = \infty$.
5. Separate real and imaginary parts.
6. Put imaginary part = 0 and find ω . Using ω find $G(j\omega)$.
7. Put real part = 0 and find ω . Using ω find $G(j\omega)$.

C. Phase margin :

1. The phase margin is that amount of the additional phase lag at the gain cross over frequency required to bring the system to the verge of instability.
2. Gain cross over frequency is the frequency at which the magnitude of open loop transfer function ($|G(j\omega)|$) is unity.
3. Phase margin is equal to 180° plus the angle of $G(j\omega)$ at the gain cross over point.

$$\phi_m = 180^\circ + \phi$$

D. Gain margin :

1. It is the reciprocal of magnitude $|G(j\omega)|$ at the frequency at which the phase angle is -180° .

Gain Margin (GM),

$$K_g = \frac{1}{|G(j\omega_c)|}$$

2. When the plot passes through the point $(-1 + j0)$, the gain margin is zero it means that the loop gain can no longer be increased because the system is at verge of instability.
3. If gain margin is positive, it means that the system is stable and negative gain margin means the system is unstable.

E. Stability :

1. To determine the stability of the system first draw the polar plot for given transfer function.
2. If the critical point $(-1 + j0)$ is within the plot, it means the system is unstable.
3. If the polar plot passes through the point $(-1 + j0)$ the system is marginally stable and if the critical point is outside the polar plot, the system is stable.

Que 4.3. Sketch the polar plot of the following function, also determine Gain Margin, Phase Margin, $H(s) = 1$

$$G(s) = \frac{1}{s(1+s)(1+2s)}$$

AKTU 2021-22, Marks 10

Answer

Given: $G(s) = \frac{1}{s(s+1)(2s+1)}$, $H(s) = 1$

To Find: Gain margin and phase margin.

To Sketch: Polar plot.

- $G(s) = \frac{1}{s(s+1)(2s+1)}$
- At $\omega = 0$, $|G(j\omega)| = \infty$, $\angle G(j\omega) = \frac{0^\circ}{+90^\circ + 0^\circ + 0^\circ} = -90^\circ$
- At $\omega = \infty$, $|G(j\omega)| = 0$, $\angle G(j\omega) = \frac{0^\circ}{90^\circ + 90^\circ + 90^\circ} = -270^\circ$
- Rotation of plot $= -270^\circ - (-90^\circ) = -180^\circ$ clockwise
- $$G(j\omega) = \frac{-j\omega(1-j\omega)(1-2j\omega)}{(j\omega)(-j\omega)(1+j\omega)(1-j\omega)(1+2j\omega)(1-2j\omega)}$$

$$= \frac{-j\omega[1-3j\omega-2\omega^2]}{\omega^2(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{-3\omega^2}{\omega^2(1+\omega^2)(1+4\omega^2)} - \frac{j\omega(1-2\omega^2)}{\omega^2(1+\omega^2)(1+4\omega^2)}$$

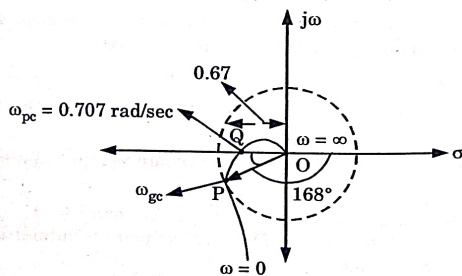


Fig. 4.3.1.

6. For imaginary part to be zero, $1 - 2\omega^2 = 0$

$$\omega^2 = \frac{1}{2} = 0.5$$

$$|G(j\omega)| = 1$$

7. $\frac{1}{\omega\sqrt{\omega^2+1}\sqrt{4\omega^2+1}} = 1$

$$4\omega^6 + 5\omega^4 + \omega^2 - 1 = 0$$

$$\therefore \omega_{gc} = 0.57$$

$$\omega = 0.707 \text{ rad/sec} = \omega_{pc}$$

8. Intersection with negative real axis,

$$\text{Point } Q = \frac{-3(0.5)}{0.5(1+0.5)(1+4 \times 0.5)} = -0.67$$

9. From Fig. 4.3.1 unit radius circle intersect polar plot at P

and $GM = 20 \log \frac{1}{OQ}$

$$PM = 180^\circ + \angle G(j\omega)|_{\omega_{gc}} = 180^\circ - 168^\circ = +12^\circ$$

ω_{gc} is the gain cross over frequency where $|G(j\omega)| = 1$.

$$GM = 20 \log \frac{1}{0.67} = 3.48 \text{ dB}$$

10. As GM and PM are positive, system is stable.

Que 4.4. Sketch the polar plot for the following transfer function

$$G(s) = (1+4s)/s^2(s+1)(2s+1)$$

AKTU 2018-19, Marks 07

Answer

1. Putting $s = j\omega$, $G(j\omega) = \frac{1+j4\omega}{(j\omega)^2(j\omega+1)(j2\omega+1)}$

2. $M = \frac{\sqrt{1+16\omega^2}}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$

3. The value of M, when $\omega = 0$, $M = \infty$, when $\omega = \infty$, $M = 0$
4. $\phi = \tan^{-1} 4\omega - 180^\circ - \tan^{-1} \omega - \tan^{-1} 2\omega$
5. The angle ϕ for different values of ω is tabulated in Table 4.4.1:

Table 4.4.1.

ω	$\tan^{-1} 4\omega$	-180°	$-\tan^{-1} \omega$	$-\tan^{-1} 2\omega$	ϕ
0	0	-180°	0	0	-180°
∞	90°	-180°	-90°	-90°	-270°

6. Polar plot makes an intercept on the negative real axis at point X.

where, Z = Number of zeros of $1 + G(s)H(s)$ in right-half s -plane.
 N = Number of clockwise encirclements of the $(-1 + j0)$ point.

P = Number of poles of $G(s)H(s)$ in the right-half s -plane.

3. If P is not zero, for a stable control system, we must have $Z = 0$ or $N = -P$ i.e., P must have counter clockwise encirclements of the $(-1 + j0)$ point.
4. If $G(s)H(s)$ does not have any poles in the right half s -plane then $Z = N$. Thus, for stability there must be no encirclement of the $(-1 + j0)$ point by $G(j\omega)H(j\omega)$.
5. The stability of system can be obtained if $(-1 + j0)$ point is enclosed by the Nyquist plot of $G(j\omega)H(j\omega)$. For stability the $(-1 + j0)$ must lies outside the graph.

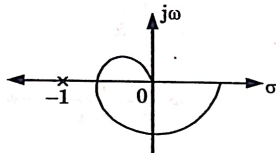


Fig. 4.6.1.

6. The encirclements of $(-1 + j0)$ point by $G(j\omega)H(j\omega)$ locus does not give proper information to detect instability in multiple loop system. The Routh Hurwitz could be applied to $1 + G(s)H(s)$ to find poles in the right-half of s -plane.
7. If the Nyquist path in the s -plane encircles Z zeros and P poles of $1 + G(s)H(s)$ and does not pass through any poles or zeros of $1 + G(s)H(s)$ as s moves in the clockwise direction along the Nyquist path, then the contour in the $G(s)H(s)$ plane encircle the $(-1 + j0)$ point $N = Z - P$ times in the clockwise direction.
8. **The three possibilities are :**
 - i. There is no encirclement of $(-1 + j0)$ point. So, system is stable if there are no poles of $G(s)H(s)$ in the right-half of s -plane.
 - ii. There is a counter clockwise encirclements of the $(-1 + j0)$. Then the system is stable if the number of counter clockwise encirclements is same as the number of poles of $G(s)H(s)$ in the right-half s -plane.
 - iii. There is a clockwise encirclement of the $(-1 + j0)$ point. Then, system is unstable.

Que 4.7. Draw the Nyquist plot for the unity feedback system whose open loop transfer function is

$$G(s)H(s) = \frac{K}{s^2(1+sT)}$$

AKTU 2018-19, Marks 07

Answer

Given : $G(s)H(s) = \frac{K}{s^2(1+sT)}$
To Find : Nyquist plot.

1. Put $s = j\omega$,

$$G(j\omega)H(j\omega) = \frac{K}{(j\omega)^2(1+j\omega T)}$$

$$2. |G(j\omega)H(j\omega)| = \frac{K}{\omega^2 \sqrt{1+\omega^2 T^2}}$$

$$3. \angle G(j\omega)H(j\omega) = -180^\circ - \angle \tan^{-1} \omega T$$

$$4. \text{ At } \omega = 0, \quad G(0)H(0) = \infty \angle -180^\circ$$

$$\text{ At } \omega = \infty, \quad G(\infty)H(\infty) = 0 \angle -270^\circ = 0 \angle 90^\circ$$

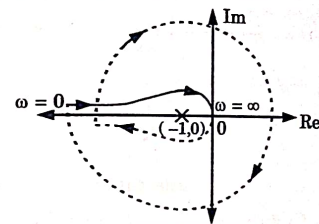


Fig. 4.7.1.

Que 4.8. Draw Nyquist plot for the open loop transfer function is

$$G(s)H(s) = 1/s(s+2).$$

AKTU 2020-21, Marks 10

Answer

Given : $G(s)H(s) = \frac{1}{s(s+2)}$
To Draw : Nyquist plot.

$$1. G(s)H(s) = \frac{1}{s(s+2)}$$

2. Put $s = j\omega$,

$$G(j\omega)H(j\omega) = \frac{1}{j\omega(j\omega+2)}$$

$$3. |G(j\omega)H(j\omega)| = \frac{1}{\omega \sqrt{\omega^2 + 4}}$$

$$\begin{aligned} \angle G(j\omega)H(j\omega) &= \frac{1}{\tan^{-1} \frac{\omega}{0} + \tan^{-1} \left(\frac{\omega}{2} \right)} \\ &= \frac{1}{90 + \tan^{-1} \frac{\omega}{2}} = -90 - \tan^{-1} \left(\frac{\omega}{2} \right) \end{aligned}$$

Table 4.8.1.

ω	M	ϕ
0	∞	-90
∞	0	-180

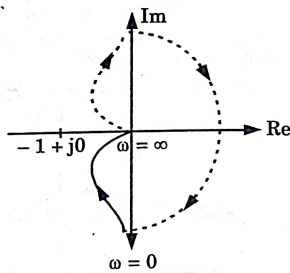


Fig. 4.8.1.

5. The point $(-1 + j0)$ is outside the Nyquist plot so there is no encirclement.
 Hence $N = 0$
 Also $P = 0$ (No poles is lying on the right hand side)
 $N = P - Z$
 $0 = 0 - Z$
 $Z = 0$

So the given system is stable.

Que 4.9. Sketch Nyquist plot for $G(s)H(s) = 6/s^2(s+2)$ comment on stability.

AKTU 2019-20, Marks 07

Answer

Given : $G(s)H(s) = 6/s^2(s+2)$

To Sketch : Nyquist plot.

1. In frequency domain the given transfer function will become,

$$G(j\omega)H(j\omega) = \frac{6}{(j\omega)^2(j\omega+2)}$$

2. Magnitude (M),

$$|G(j\omega)H(j\omega)| = \frac{6}{\omega^2 \sqrt{\omega^2 + 4}}$$

3. Phase angle (ϕ),

$$\angle G(j\omega)H(j\omega) = \angle -180 - \tan^{-1} \frac{\omega}{2}$$

4. For various value of ω , the value of M and ϕ are :

Table 4.9.1.

ω	M	ϕ
0	∞	-180°
∞	0	-270°

5. The corresponding Nyquist plot is shown in Fig. 4.9.1.

6. For stability,
 where,
 $N = P - Z$
 $N = 2$ (Number of encirclement)
 $P = 0$ (Number of poles on right-hand side)
 $2 = 0 - Z$
 $Z = 2$ (Number of zeros on right-hand side)

So the system is unstable because its value lies in the right half s-plane.

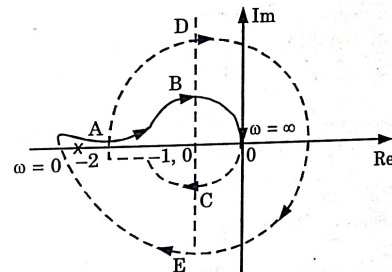


Fig. 4.9.1.

Que 4.10. Sketch the Nyquist plot for the system with open loop transfer function

$$G(s)H(s) = \frac{60}{(s+1)(s+2)(s+5)}$$

and comment on stability.

Answer

Given : $G(s)H(s) = \frac{60}{(s+1)(s+2)(s+5)}$

To Sketch : Nyquist plot.

- Put $s = j\omega$, $G(j\omega)H(j\omega) = \frac{60}{(j\omega + 1)(j\omega + 2)(j\omega + 5)}$
- Magnitude, $M = |G(j\omega)H(j\omega)| = \frac{60}{(\sqrt{\omega^2 + 1})(\sqrt{\omega^2 + 4})(\sqrt{\omega^2 + 25})}$
- Phase $\phi = \angle G(j\omega)H(j\omega)$
 $= -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$

Table 4.10.1.

ω	M	ϕ
0	6	0°
∞	0	$90^\circ (-270^\circ)$

- $N = P - Z$
 $P = 0$ (Number of poles on right hand side)
 $N = 0$ (Number of encirclement of $-1 + j0$ point)
 $Z = 0$ (Number of zero on right hand side).
 So the given system is stable system.

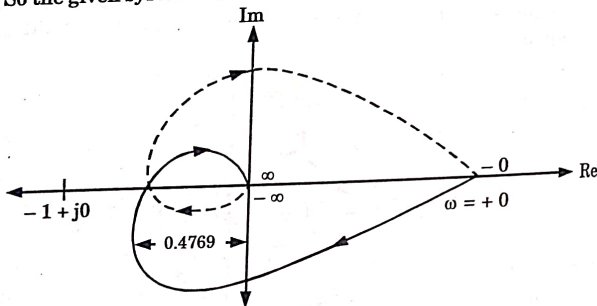


Fig. 4.10.1.

Que 4.11. Draw the complete Nyquist plot for a unity feedback system having the open loop function, from this plot obtain all the information regarding stability.

$$G(s)H(s) = k/s(s+3)(s+5) \quad \text{AKTU 2021-22, Marks 10}$$

Answer

The procedure is same as Q. 4.10, Page 4-12C, Unit-4.

- $P = 0$ (Number of poles on R-H plane)
 $N = -1$ [Because contour has infinite radius, so it encircle $(-1, 0)$ point]

$$\begin{aligned} N &= P - Z \\ -1 &= 0 - Z \\ Z &= 1 \end{aligned}$$

So the given system is unstable

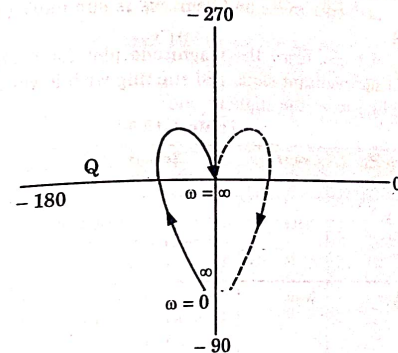


Fig. 4.11.1.

PART-4

Determination of Gain and Phase Margin From Bode and Nyquist Plots.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.12. Write procedure to draw Bode plot from given open loop transfer function.

Answer

- Bode plot :** In this plot we have two plots namely Magnitude plot; which is drawn in between magnitude of transfer function in dB and $\log_{10} \omega$ (ω is frequency) and phase plot; which is drawn in between phase angle of transfer function in degree and $\log_{10} \omega$ generally.
- Procedure to draw Bode plot :**

Step 1 :

Replace $s = j\omega$ from open loop transfer function $G(s)$ to have $G(j\omega)$.

Step 2 :
Change all the factor of $G(j\omega)$ in standard form and make a table with remarks for both magnitude plot and phase plot.
Some standard results and remarks is summarized here :

Step 3 :
From the table, draw the magnitude plot for each of the factor by calculating resultant slope and starting with lowest corner frequency and ending with the highest one.

Table 4.12.1.

S. No.	Factors of $G(j\omega)$	Corner Frequencies	Remarks for Magnitude Plot	Remarks for Phase Plot
1.	$\frac{K}{j\omega}$	None	Straight line of constant slope - 20 dB/decade passing through $20 \log K$ at $\omega = 1$	Constant - 90°
2.	$Kj\omega$	None	Slope = + 20 dB/decade straight line of constant slope - 20 dB/decade through $20 \log K$ at $\omega = 1$	Constant + 90°
3.	$\frac{1}{1 + j\omega T_1}$	$\omega_{c1} = \frac{1}{T_1}$	Straight line of slope - 20 dB/decade for $\omega > \omega_{c1}$	Phase angle = $-\tan^{-1}(\omega T)$
4.	$(1 + j\omega T_1)$	$\omega_{c2} = \frac{1}{T_2}$	Straight line of slope + 20 dB/decade for $\omega > \omega_{c2}$	Phase angle = $-\tan^{-1}(\omega T)$
5.	$\frac{1}{1 + 2j\zeta\left(\frac{\omega}{\omega_n}\right) + \left(\frac{\omega}{\omega_n}\right)^2}$	$\omega_{c3} = \omega_n$	Straight line of slope - 40 dB/decade for $\omega > \omega_{c3}$	Phase angle = $-\tan^{-1}\left[\frac{\zeta(\omega/\omega_n)}{1 - (\omega/\omega_n)^2}\right]$
6.	$1 + 2j\zeta\left(\frac{\omega}{\omega_n}\right) + \left(\frac{\omega}{\omega_n}\right)^2$	$\omega_{c4} = \omega_n$	Straight line of slope + 40 dB/decade for $\omega > \omega_{c4}$	Phase angle = $+\tan^{-1}\left[\frac{\zeta(\omega/\omega_n)}{1 - (\omega/\omega_n)^2}\right]$

Step 4 :

Add all the expression of phase angle to have different values of ϕ (resultant phase).

Step 5 :

Draw ϕ with $\log_{10}\omega$ to have phase plot.

Que 4.13. Explain gain margin and phase margin and how to determine them using Bode plot.

Answer

A. Gain Margin (GM) :

1. It is the margin in gain which is allowed till the system reaches on the verge of instability.
2. Mathematically, gain margin is reciprocal of magnitude of $G(j\omega)H(j\omega)$ at phase crossover frequency.

$$GM = 1 / |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$$

B. Phase Margin (PM) : It is the amount of additional phase lag which can be introduced in the system till it reaches on the verge of instability.

Mathematically, $PM = 180^\circ + \angle G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$

C. Calculation of GM and PM from Bode plot :

$$GM = -20 \log |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} \text{ dB}$$

$$= 0 \text{ dB} - \log |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} \text{ dB}$$

1. $20 \log |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$ can be directly known from magnitude plot.

$$PM = 180^\circ + \angle G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$$

2. Now,

$\angle G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$ can be directly known on phase angle plot.

3. **Gain crossover frequency (ω_{gc}) :**
Frequency at which magnitude of $G(j\omega)H(j\omega)$ is unity.
4. **Phase crossover frequency (ω_{pc}) :**
Frequency at which phase angle of $G(j\omega)H(j\omega)$ is -180° .
5. Stable system = PM and GM positive.
Unstable system = PM and GM negative
Marginally stable = PM = 0, GM = 0

Que 4.14. A unity feedback control systems has :

$$G(s) = \frac{40}{s(s+2)(s+5)}$$

Draw the Bode Plot. Find Gain Margin.

Answer

Given : $G(s) = \frac{40}{s(s+2)(s+5)}$

To Draw : Bode plot.

To Find : Gain margin.

1. **Magnitude :**

$$G(s) = \frac{40}{s(s+2)(s+5)} = \frac{40}{s \times 2 \left(1 + \frac{s}{2}\right) 5 \left(1 + \frac{s}{5}\right)} = \frac{4}{s \left(1 + \frac{s}{2}\right) \left(1 + \frac{s}{5}\right)}$$

First corner frequency (pole) (ω_1) = 2 rad/s

Second corner frequency (pole) (ω_2) = 5 rad/s

2. **Phase angle :** $G(s) = \frac{4}{s \left(1 + \frac{s}{2}\right) \left(1 + \frac{s}{5}\right)}$

Put

$$s = j\omega$$

$$G(j\omega) = \frac{4}{j\omega(1 + 0.5j\omega)(1 + 0.2j\omega)}$$

$$\phi = -90^\circ - \tan^{-1} 0.5\omega - \tan^{-1} 0.2\omega$$

3. Magnitude plot :

Table 4.14.1.

S.No.	Factor	Corner frequency characteristics	Asymptotic log magnitude
1.	$1/s$	None	Straight line of constant slope (-20 dB/dec) passing through at $\omega = 1$
2.	$1/(1 + 0.5s)$	$\omega_1 = 2$	Straight line of constant slope (-20 dB/dec) originating from $\omega_1 = 2$
3.	$1/(1 + 0.2s)$	$\omega_2 = 5$	Straight line of constant slope (-20 dB/dec) originating from $\omega_2 = 5$
4.	4	None	Straight line of constant slope of 0 dB/dec starting from $20 \log 4 = 12 \text{ dB}$ point

4. Phase plot :

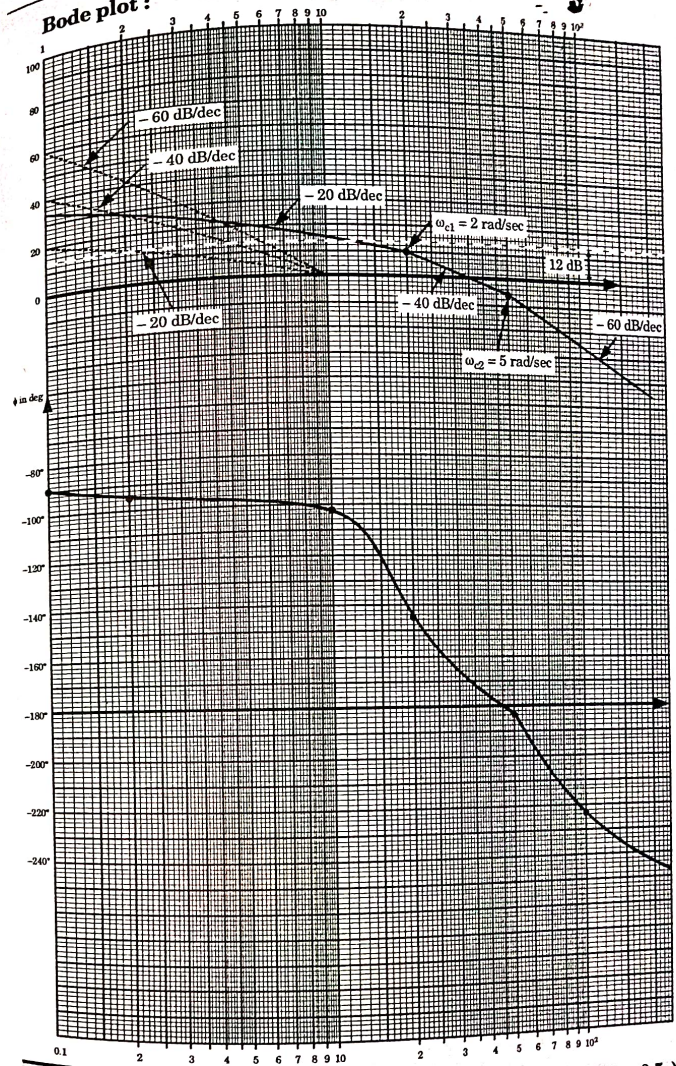
Table 4.14.2.

S.No.	ω (rad/sec)	ϕ (degrees)
1.	0	-90°
2.	0.2	-98°
3.	1	-127.874°
4.	2	-156.8°
5.	5	-203.198°
6.	10	-232.125°
7.	15	-243.97°

Result :

- Gain crossover frequency = 2.8 rad/s
- Phase cross over frequency = 3.3 rad/s
- Gain margin = 3 dB
- Phase margin = 8°.

Bode plot :

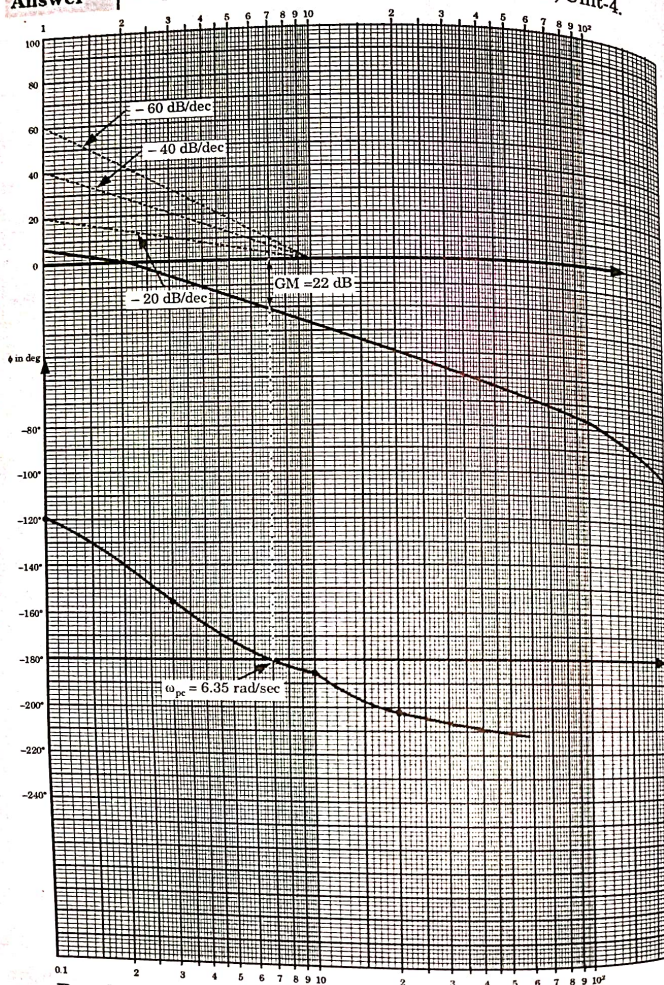


Que 4.15. For the given transfer function $G(s)H(s) = \frac{2}{s(1 + 0.5s)}$ (1+0.05s). Determine phase crossover frequency and gain margin.

AKTU 2018-19, Marks 07

Answer

The procedure is same as Q. 4.14, Page 4-16C, Unit-4.

**Results from bode plot :**

1. Phase crossover frequency (ω_p) = 6.35 rad/sec
2. Gain margin = 22 dB

Que 4.16. Draw the bode plot of the unity feedback control system having open loop transfer function.

$$G(s) = \frac{10}{s(1 + 0.02s)(1 + 0.2s)}$$

Also determine GM and discuss the stability of closed loop system.

Answer

Given : $G(s) = \frac{10}{s(1 + 0.02s)(1 + 0.2s)}$

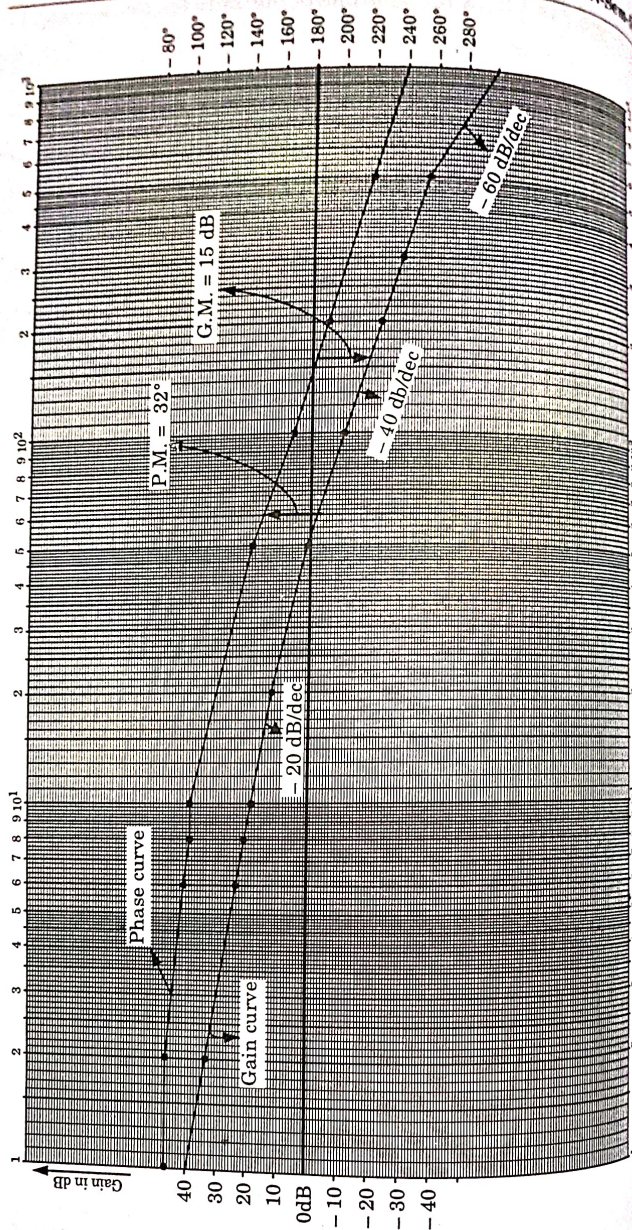
To Find : Gain Margin, Stability

To Draw : Bode a Plot.

1. Put $s = j\omega$, $G(j\omega) = \frac{10}{j\omega(1 + 0.02j\omega)(1 + 0.2j\omega)}$
 $|G(j\omega)| = \frac{10}{\omega \sqrt{1 + (0.02\omega)^2} \sqrt{1 + (0.2\omega)^2}} \dots(4.16.1)$
2. Magnitude in dB = $20 \log |G(j\omega)|$
 $= 20 \log 10 - 20 \log [\omega \sqrt{1 + (0.02\omega)^2} \sqrt{1 + (0.2\omega)^2}] \dots(4.16.2)$
3. Angle of $G(j\omega)$, $\angle G(j\omega) = -90^\circ - \tan^{-1}(0.02\omega) - \tan^{-1}(0.2\omega) \dots(4.16.3)$

Table 4.16.1.

ω rad/s	$\angle G(j\omega)$	Magnitude in dB
$\omega = 0.1$	-91.26°	39.00
0.2	-90.22°	33.00
0.5	-96.28°	25.90
0.7	-97.96°	23.30
0.9	-100.20°	20.70
1.0	-102.45°	19.80
5.0	-140.71°	02.96
10.0	-164.74°	-7.16
20.0	-187.76°	-18.05
50.0	-219.30°	-37.35
100.0	-240.57°	-53.42



$$GM = 15 \text{ dB}$$

$$PM = 32^\circ$$

- 4.
- 5.
6. Gain margin and phase margin both are positive in nature, then the given closed loop system is stable.

Que 4.17. Draw Bode Plot of unity feedback control system having

$$OLTF G(s) = \frac{10}{s(1 + 0.2s)(1 + 0.2s)}$$

Determine GM, PM, gain cross over frequency, phase cross over frequency and discuss stability of closed loop system.

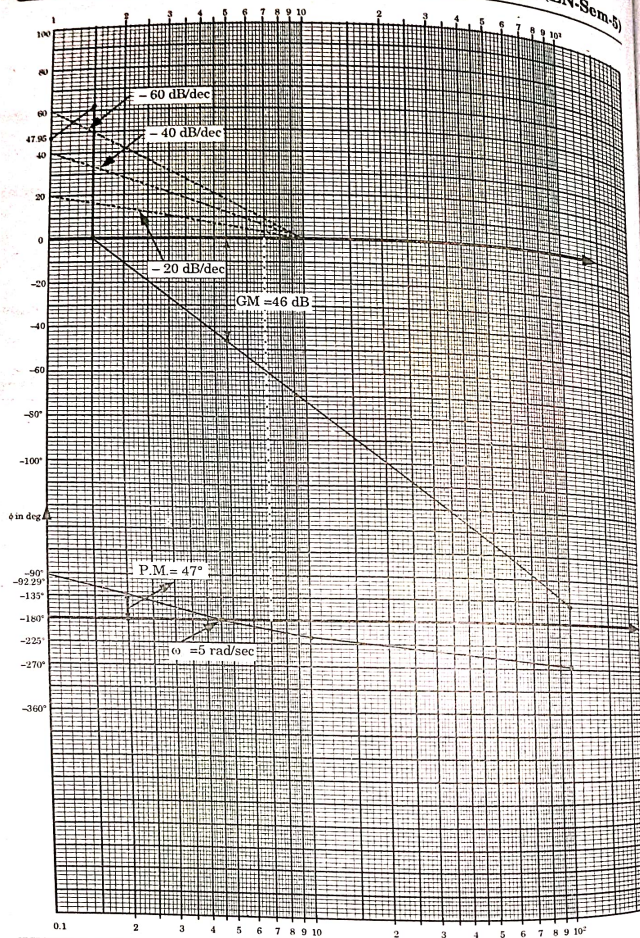
AKTU 2019-20, Marks 07

Answer

The procedure is same as Q. 4.16, Page 4-20C, Unit-4.

Results from bode plot :

1. Gain margin = 46 dB
2. Phase margin = 47°
3. Gain crossover frequency = 0.2 rad/sec
4. Phase crossover frequency (ω_{pc}) = 5 rad/sec
5. **Stability :** The gain margin and phase margin both are positive therefore, the system is stable.



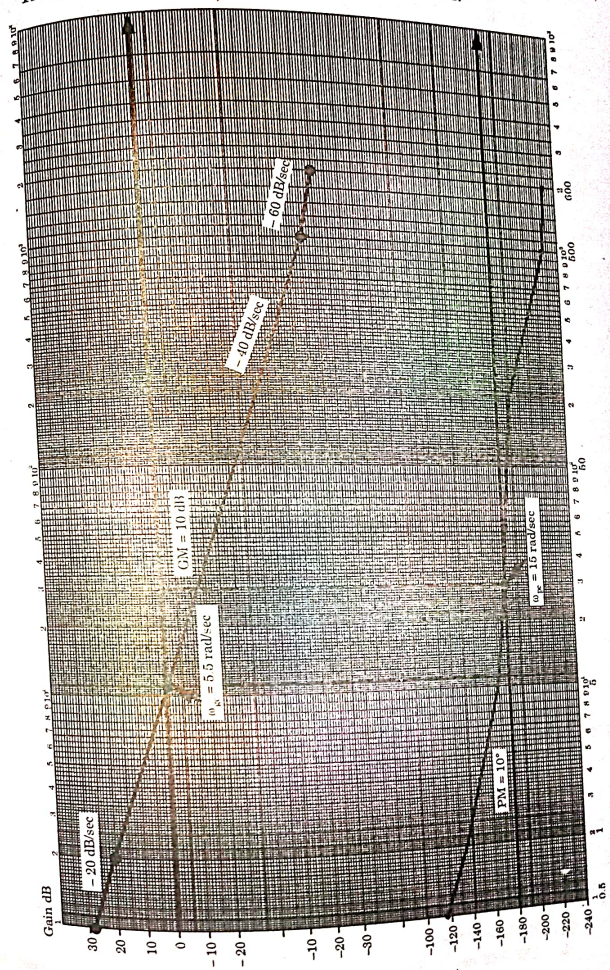
Que 4.18. Sketch Bode plot for the following transfer function and determine the gain cross over frequency and phase cross over frequency, comment on stability.

$$G(s)H(s) = 10/s(1+s)(1+0.002s)$$

AKTU 2021-22, Marks 10

Answer

The procedure is same as Q. 4.16, Page 4-20C, Unit-4.



Results from Bode plot :

1. Gain crossover frequency, $\omega_{gc} = 5.5$ rad/sec
2. Phase crossover frequency, $\omega_{pc} = 15$ rad/sec
3. Gain margin, $GM = 10$ dB
4. Phase margin, $PM = 10^\circ$

Because the GM and PM both are positive. So the given system is stable.

PART-5**Correlation Between Time and Frequency Responses.****Questions-Answers****Long Answer Type and Medium Answer Type Questions**

Que 4.19. Explain various frequency domain specifications.

Answer

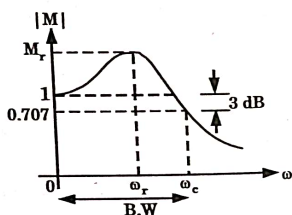
1. **Resonant peak (M_r)** : Resonance peak is the peak value of magnitude of closed loop frequency response.

$$\text{For second order, } M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

2. **Resonance frequency** : The frequency at which resonant peak occurs is called resonant frequency.

$$\text{For second order, } \omega_r = \omega_n \sqrt{1-2\xi^2}$$

3. **Bandwidth** : It is the range of frequency up to ω_c , which is cut off frequency. At ω_c the magnitude of closed loop transfer function is 3 dB down from its zero frequency level.

**Fig. 4.19.1.**

4. **Cut-off frequency** : The frequency at which magnitude is 3 dB below its zero frequency value is called cut-off frequency (ω_c).
5. **Cut-off rate** : The cut-off rate is the frequency rate at which the magnitude ratio decreased beyond the cut-off frequency ω_c .

Que 4.20. Establish the correlation between time response and frequency response analysis and suitably explain with diagrams.

AKTU 2018-19, Marks 07

OR

Explain the strengths of frequency response approach; establish correlation between frequency domain response and time domain response.

AKTU 2019-20, Marks 07

Answer

Strengths of frequency response (Advantages) : Refer Q. 4.1, Page 4-2C, Unit-4.

Correlation between time response and frequency response :

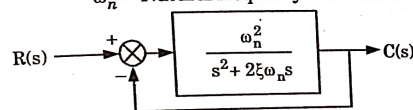
1. For 2nd order system, the transfer function is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where,

ξ = Damping factor

ω_n = Natural frequency of oscillations

**Fig. 4.20.1.**

2. Closed loop frequency response is,

$$\begin{aligned} \frac{C(j\omega)}{R(j\omega)} &= T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2} \\ &= \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n(j\omega) + \omega_n^2} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2j\xi\left(\frac{\omega}{\omega_n}\right)} \\ &= \frac{1}{(1-u^2) + j2\xi u} \end{aligned} \quad \dots(4.20.1)$$

where $u = \omega/\omega_n$, normalized driving frequency.

$$\therefore |T(j\omega)| = M = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \quad \dots(4.20.2)$$

$$\text{and } \angle T(j\omega) = \phi = -\tan^{-1} \frac{2\xi u}{1-u^2} \quad \dots(4.20.3)$$

3. The steady state output is

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \sin\left(\omega t - \tan^{-1} \frac{2\xi u}{1-u^2}\right)$$

4. The frequency where M has a peak value is called the resonant frequency. At this frequency the slope of magnitude curve is zero.

If

ω_r = Resonant frequency.

$u_r = \omega_r / \omega_n$ is normalized resonant frequency.

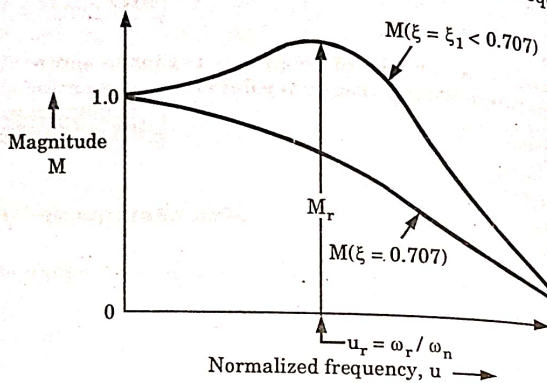


Fig. 4.20.2. Frequency response magnitude characteristics.

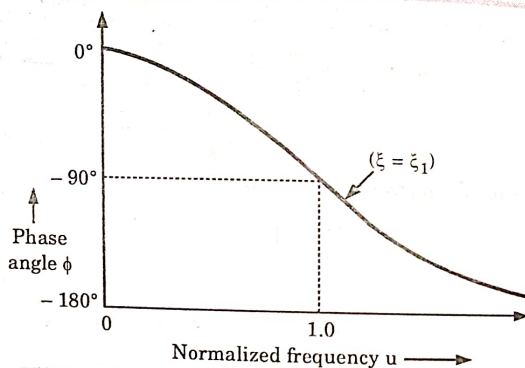


Fig. 4.20.3. Frequency response phase characteristic.

$$\left. \frac{dM}{du} \right|_{u=u_r} = -\frac{1}{2} \frac{[-4(1-u_r^2)u_r + 8\xi^2 u_r]}{[(1-u_r^2)^2 + (2\xi u_r)^2]^{3/2}} = 0$$

$$-4(1-u_r^2)u_r + 8\xi^2 u_r = 0$$

$$-4u_r(1-u_r^2 - 2\xi^2) = 0$$

$$\therefore u_r = \sqrt{1-2\xi^2}$$

$$\omega_r = \omega_n \sqrt{1-2\xi^2}$$

...(4.20.4)

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad \dots(4.20.5)$$

5. The phase angle ϕ of $T(j\omega)$ at resonant frequency is

$$\phi_r = -\tan^{-1} \left[\frac{\sqrt{1-2\xi^2}}{\xi} \right]$$

Bandwidth :

1. The frequency at which M has value of $\frac{1}{\sqrt{2}}$ and is called cut-off frequency ω_c . The signal frequencies above cut-off are attenuated.
2. The range of frequencies for which $M \geq \frac{1}{\sqrt{2}}$ is known as bandwidth ω_b . The low pass filters has bandwidth equal to cut-off. ω_b indicates the noise filtering characteristics of the system.

3. Normalized bandwidth, $u_b = \omega_b / \omega_n$

$$\therefore M = \frac{1}{\sqrt{(1-u_b^2)^2 + (2\xi u_b)^2}} = \frac{1}{\sqrt{2}}$$

$$\therefore u_b = [1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}]^{\frac{1}{2}}$$

$$\omega_b = \omega_n [1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}]^{\frac{1}{2}}$$

4. The damped frequency of oscillations ω_d and peak overshoot M_p of the step response for $0 \leq \xi \leq 1$ are

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$M_p = \exp \left[\frac{-\pi\xi}{\sqrt{1-\xi^2}} \right]$$

Que 4.21. The forward path transfer function of unity feedback control system is $G(s) = 100/s(s + 6.45)$. Find the resonance peak M_r , resonant frequency ω_r and bandwidth of the closed loop system.

AKTU 2018-19, Marks 07

Answer

1.

$$G(s) = \frac{100}{s(s + 6.45)} \text{ and } H(s) = 1$$

2.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{100/(s(s + 6.45))}{1 + 100/(s(s + 6.45))}$$

$$= \frac{100}{s^2 + 6.45s + 100}$$

3. Compare eq. (1) with $\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$... (4.21.1)
- so, $\omega_n = \sqrt{100} = 10 \text{ rad/sec}$
 $2\xi\omega_n = 6.45 \therefore \xi = 0.32$
4. Resonant frequency, $\omega_r = \omega_n \sqrt{1 - 2\xi^2} = 8.91 \text{ rad/sec}$
5. Resonant peak, $M_r = \frac{1}{2\xi\sqrt{1 - \xi^2}} = \frac{1}{2 \times 0.32 \sqrt{1 - 0.32^2}} = 1.65$
6. Bandwidth, $\omega_b = \omega_n \sqrt{1 - 2\xi^2 + (2 - 4\xi^2 + 4\xi^4)^{1/2}} = 14.39 \text{ rad/sec}$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

- Q. 1. What do you understand by frequency response? What are its various methods? Discuss their advantages, disadvantages and limitations.

Ans. Refer Q. 4.1, Unit-4.

- Q. 2. Sketch the polar plot of the following function, also determine Gain Margin, Phase Margin, $H(s) = 1$

$$G(s) = \frac{1}{s(1+s)(1+2s)}$$

Ans. Refer Q. 4.3, Unit-4.

- Q. 3. Sketch the polar plot for the following transfer function
 $G(s) = (1 + 4s)/s^2(s + 1)(2s + 1)$

Ans. Refer Q. 4.4, Unit-4.

- Q. 4. Sketch Nyquist plot for $G(s)H(s) = 6/s^2(s + 2)$ comment on stability.

Ans. Refer Q. 4.9, Unit-4.

- Q. 5. Sketch the Nyquist plot for the system with open loop transfer function

$$G(s)H(s) = \frac{60}{(s + 1)(s + 2)(s + 5)} \text{ and comment on stability.}$$

Ans. Refer Q. 4.10, Unit-4.

- Q. 6. A unity feedback control systems has :

$$G(s) = \frac{40}{s(s + 2)(s + 5)}$$

Draw the Bode Plot. Find Gain Margin.

Ans. Refer Q. 4.14, Unit-4.

- Q. 7. Sketch Bode plot for the following transfer function and determine the gain cross over frequency and phase cross over frequency, comment on stability.

$$G(s)H(s) = 10/s(1 + s)(1 + 0.002s)$$

Ans. Refer Q. 4.18, Unit-4.

- Q. 8. Explain the strengths of frequency response approach; establish correlation between frequency domain response and time domain response.

Ans. Refer Q. 4.20, Unit-4.

- Q. 9. The forward path transfer function of unity feedback control system is $G(s) = 100/s(s + 6.45)$. Find the resonance peak M_r , resonant frequency ω_r , and bandwidth of the closed loop system.

Ans. Refer Q. 4.21, Unit-4.



5

UNIT

Introduction to Design

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- Part-1 :** The Design Problems and 5-2A to 5-16A
Preliminary Considerations of Lead, Lag and Lead-Lag Compensation Networks, Design of Closed Loop System using Compensation Techniques in Time and Frequency Domain
- Part-2 :** State Space Technique : The 5-16A to 5-17A
Concept of State and Space
- Part-3 :** State Space Model of Physical 5-17A to 5-26A
System, Conversion of State Space to Transfer Function Model and Vice-Versa
- Part-4 :** State Transition Matrix 5-26A to 5-29A
- Part-5 :** Concept of Controllability and 5-29A to 5-36A
Observability and Their Testing

5-1 C (EN-Sem-5)

5-2 C (EN-Sem-5)

Introduction to Design

PART-1

The Design Problems and Preliminary Considerations of Lead, Lag and Lead-Lag Compensation Networks, Design of Closed Loop System using Compensation Techniques in Time and Frequency Domain.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. Define Phase lead compensation technique. How would you draw its pole zero configuration ?

Answer

A. Lead compensation :

- Fig. 5.1.1 shows a phase-lead network where in the phase of output voltage leads the phase of input voltage for sinusoidal input.

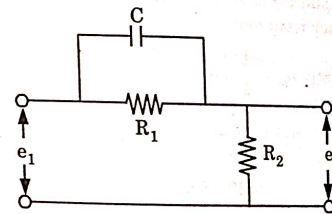


Fig. 5.1.1. Phase lead network.

- The transfer function of a phase lead network,

$$\frac{E_o(s)}{E_i(s)} = \frac{\alpha(1 + sT)}{(1 + \alpha sT)} \quad \dots(5.1.1)$$

$$\frac{E_o}{E_i} = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

where,

$$\alpha < 1$$

$$\alpha = \frac{R_2}{R_1 + R_2}$$

and $T = R_1 C$

3. The transfer function given by eq. (5.1.1) can be expressed in sinusoidal form as

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{\alpha(1 + j\omega T)}{(1 + j\omega\alpha T)} \quad \dots(5.12)$$

4. The pole zero configuration of eq. (5.1.2) shown in Fig. 5.1.2.

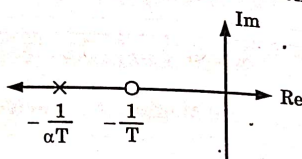


Fig. 5.1.2. Pole zero configuration.

B. Bode plot of lead compensator :

1. The two corner frequencies are

$$\omega_{c1} = \frac{1}{T}, \text{ lower corner frequency}$$

$$\omega_{c2} = \frac{1}{\alpha T}, \text{ upper corner frequency}$$

2. The maximum phase lead ϕ_m occurs at mid-frequency ω_m between upper and lower corner frequencies

$$\therefore \log_{10} \omega_m = \frac{1}{2} \left[\log_{10} \left(\frac{1}{T} \right) + \log_{10} \left(\frac{1}{\alpha T} \right) \right]$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

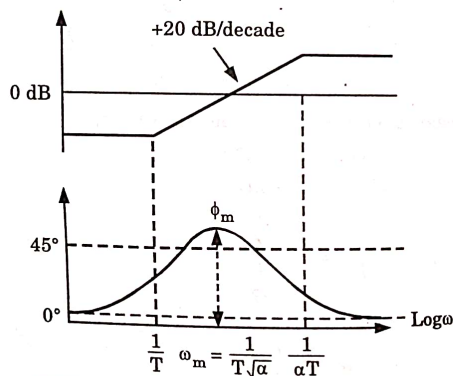


Fig. 5.1.3. Bode plot of lead compensator.

3. The phase angle $\angle E_o(j\omega)/E_i(j\omega)$ can be calculated as

$$\angle \frac{E_o(j\omega)}{E_i(j\omega)} = \tan^{-1}(\omega T) - \tan^{-1}(\omega\alpha T)$$

4. At $\omega = \omega_m = \frac{1}{T\sqrt{\alpha}},$

The phase angle is

$$\begin{aligned} \phi_m &= \tan^{-1} \left[\frac{1}{T\sqrt{\alpha}} T \right] - \tan^{-1} \left[\frac{1}{T\sqrt{\alpha}} \alpha T \right] \\ &= \tan^{-1} \left[\frac{1}{\sqrt{\alpha}} \right] - \tan^{-1} [\sqrt{\alpha}] \end{aligned}$$

$$\tan \phi_m = \frac{\frac{1}{\sqrt{\alpha}} - \sqrt{\alpha}}{1 + \frac{1}{\sqrt{\alpha}} \sqrt{\alpha}}$$

$$\tan \phi_m = \frac{1 - \alpha}{2\sqrt{\alpha}}$$

and $\sin \phi_m = \frac{1 - \alpha}{1 + \alpha}$

Que 5.2. Explain the Bode plot method of designing lead compensator.

OR

What are the effects of lead compensation?

Answer

A. Design procedure of lead compensator:

Step 1 : Consider equation

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

Let $K_c \alpha = K = \text{DC gain}$

$$G_c(s) = K \frac{Ts + 1}{\alpha Ts + 1}$$

This control system is shown in Fig. 5.2.1.

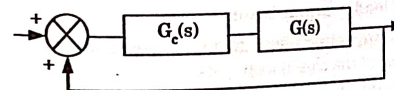


Fig. 5.2.1.

The open loop transfer function is :

$$G_c(s) G(s) = \frac{K(Ts+1)}{\alpha Ts+1} G(s)$$

$$= \frac{Ts+1}{\alpha Ts+1} KG(s) = \frac{Ts+1}{\alpha Ts+1} G_1(s)$$

where, $G_1(s) = KG(s)$

Now, determine gain K to satisfy the requirement on the given static error coefficient.

Step 2 : Using above value of K , find phase margin from Bode plot of $G_1(j\omega)$ which is the gain adjusted but uncompensated system. Generally phase margin is specified in design problems.

Step 3 : Determine the necessary phase lead (ϕ_m) required to be added by the following equation

$$\phi_m = \phi_1 - \phi_2 + \phi'$$

where,

ϕ_1 = Specified phase margin

ϕ_2 = Phase margin obtained in step 2

ϕ' = Safety margin.

Step 4 : Determine the value of the attenuation factor (α) using equation

$$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

Step 5 : Determine the frequency where the magnitude of uncompensated system $G_1(j\omega)$ is equal to $-20 \log (1/\sqrt{\alpha})$ dB. This is the new gain crossover frequency which corresponds to

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

The maximum phase shift (ϕ_m) occurs at this frequency.

Step 6 : Determine the corner frequencies of the lead compensator as:

Zero of lead compensator $\omega_{c1} = 1/T$

Pole of lead compensator $\omega_{c2} = 1/\alpha T$

Step 7 : Calculate constant K_c from value of K determined in step 1 and that of α determined in step 4, by using equation $K_c = K\alpha$.

Step 8 : Determine gain margin (G.M.). If it is not satisfactory and not according to design requirements repeat the design process by modifying the pole-zero location of the lead compensator until the satisfactory result is obtained.

B. Effects of lead compensation :

1. The lead compensator adds a dominant zero and a pole. This increases the damping of the closed loop system.
2. It improves the phase margin of the closed loop system.
3. It increases bandwidth of the closed loop system.

Que 5.3. Design a lead compensator for a system whose open loop transfer function is

$$G(s)H(s) = 4/s(s+2)$$

It will fulfill following requirement

- i. Static velocity error constant = 20 sec^{-1}
- ii. PM at least 50°
- iii. GM at least 10 db

AKTU 2021-22, Marks 10

Answer

Step 1 : Assume a lead compensator as,

$$G(s) = K_c \alpha \frac{(1+Ts)}{(1+\alpha Ts)} = \frac{K(1+Ts)}{(1+\alpha Ts)}$$

and

$$G_1(s) = K G(s) = \frac{4K}{s(s+2)}$$

$$K_v = 20 = \lim_{s \rightarrow 0} s G(s) H(s) = \lim_{s \rightarrow 0} s \frac{4K}{s(s+2)} \times \frac{(1+Ts)}{(1+\alpha Ts)}$$

$$20 = 2K \text{ i.e., } K = 10$$

$$G_1(s) = \frac{4 \times 10}{s(s+2)} = \frac{40}{s(s+2)}$$

Step 2 : Thus line of slope -20 dB/sec till $\omega_c = 2$ and line of slope -40 dB/sec from 2 onwards.

Sketch the bode plot of $G_1(s)$

Phase angle table :

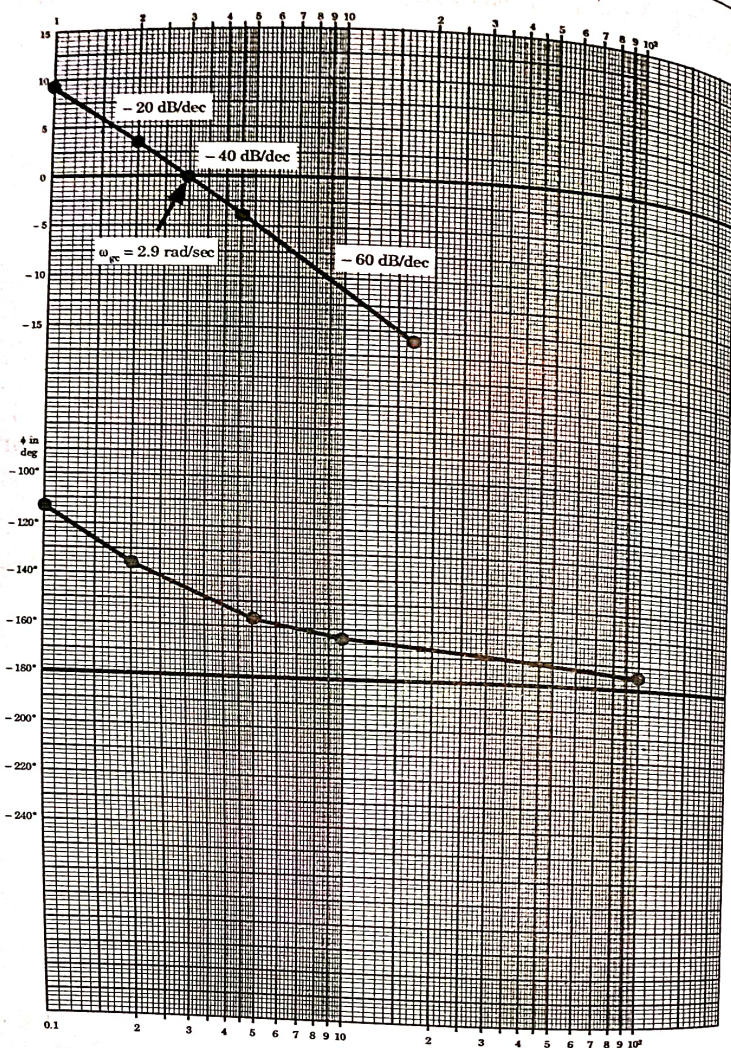
$$G_1(j\omega) = \frac{40}{j\omega(j\omega+2)}$$

Phase Plot :

Phase angle, $\phi = -90^\circ - \tan^{-1} 0.5 \omega$

S.No.	ω	ϕ
1.	1	-116.56°
2.	2	-135°
3.	5	-158.2°
4.	10	-163.7°
5.	100	-178.8°

From the Bode Plot :



$$PM = 18^\circ, GM = \infty$$

Step 3 : Given phase margin,

$$\phi_s = 50^\circ$$

5-8 C (EN-Sem-5)

Introduction to Design

Maximum phase lead angle

$$\phi_m = \phi_s - \phi_1 + \varepsilon$$

$$= 50 - 18 + 5 = 37^\circ$$

(Let $\varepsilon = 5^\circ$)

Step 4 : $\sin \phi_m = \frac{1 - \alpha}{1 + \alpha}$ i.e., $\sin 37 = \frac{1 - \alpha}{1 + \alpha} = 0.6$

$$0.6(1 + \alpha) = 1 - \alpha$$

$$\alpha = 0.25$$

Step 5 : $-10 \log \left(\frac{1}{\alpha} \right) = -6.02$ dB (for $\alpha = 0.25$)

From Bode Plot find frequency at which gain of the uncompensated system is -6.02 dB, this is ω_m .

From Bode Plot, $\omega_m = 8.8$ rad/sec = 9 rad/sec

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \text{ i.e., } \frac{1}{T} = 4.5$$

Step 6 : Two corner frequencies of the lead compensator are,

$$\omega_{c1} = \frac{1}{T} = 4.5 \text{ and } \omega_{c2} = \frac{1}{\alpha T} = 18$$

Step 7 :

$$K = K_{ca}$$

$$K_c = \frac{K}{\alpha} = \frac{10}{0.25} = 40$$

$$G_c(s) = 40 \times 0.25 \frac{(1 + 0.22s)}{(1 + 0.055s)} = \frac{10(1 + 0.22s)}{(1 + 0.055s)}$$

This is the designed lead compensator

$$G_c(s) G(s) = \frac{10(1 + 0.22s)}{(1 + 0.055s)} \times \frac{4}{s(s + 2)}$$

$$= \frac{40(1 + 0.22s)}{s(s + 2)(1 + 0.055s)}$$

$$= \frac{20(1 + 0.22s)}{s(1 + 0.5s)(1 + 0.055s)}$$

Que 5.4. Explain the lag compensation.

AKTU 2018-19, Marks 07

OR

Define phase-lag compensation techniques with the help of R-C network. Also draw its pole zero configuration.

Answer

1. In phase lag compensating network, the phase of output voltage lags the phase of input voltage for sinusoidal inputs. It is a simple and commonly used network.

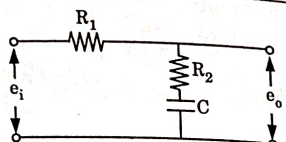


Fig. 5.4.1. Phase-lag network

2. The transfer function of phase-lag network is shown in Fig. 5.4.1,

$$\frac{E_o(s)}{E_i(s)} = \frac{1+sT}{1+s\beta T} \quad \dots(5.4.1)$$

where $\beta > 1$, $\beta = \frac{R_1 + R_2}{R_2}$

and $T = R_2 C$

3. The transfer function given by eq. (5.4.1) can be expressed in sinusoidal form as

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{1+j\omega T}{1+j\omega\beta T} \quad \dots(5.4.2)$$

4. Bode plot for transfer function of eq. (5.4.2) is shown in Fig. 5.4.2.

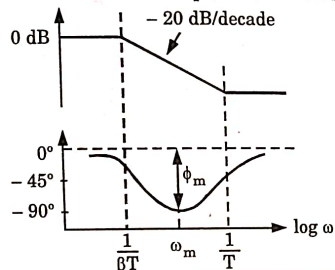


Fig. 5.4.2. Bode plot of lag compensator.

5. The two corner frequencies are $\omega = \frac{1}{T}$, upper corner frequency for

zero at $s = -\frac{1}{T}$, $\omega = \frac{1}{\beta T}$, lower corner frequency for a pole at $s = -\frac{1}{\beta T}$

6. The maximum phase-lag, ϕ_m occurs at mid frequency ω_m between upper and lower corner frequencies.

$$\therefore \log_{10} \omega_m = \frac{1}{2} \left[\log \left(\frac{1}{\beta T} \right) + \log \left(\frac{1}{T} \right) \right]$$

$$\therefore \omega_m = \frac{1}{\sqrt{\beta T}}$$

7. The phase angle $\angle E_o(j\omega)/E_i(j\omega)$ calculated as

$$\angle \frac{E_o(j\omega)}{E_i(j\omega)} = \tan^{-1}(\omega T) - \tan^{-1}(\omega\beta T)$$

At $\omega = \omega_m = \frac{1}{\sqrt{\beta T}}$, the phase angle is ϕ_m :

$$\tan \phi_m = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\sin \phi_m = \frac{1-\beta}{1+\beta}$$

8. Pole-zero configuration is shown in Fig. 5.4.3.

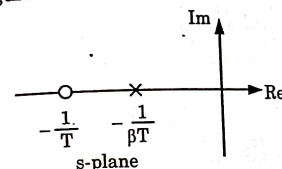


Fig. 5.4.3. Pole zero configuration.

Que 5.5. What is lag compensator? What are the characteristics of lag compensation? Explain the frequency response of lag compensator.

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Answer

A. **Lag compensator**: Refer Q. 5.4, Page 5-8C, Unit-5.

B. **Characteristics of lag compensation**:

1. Lag compensator allows high gain at low frequencies thus it is basically a low pass filter. Hence it improves the steady state performance.
2. A lag compensator is to provide attenuation in the high frequency range to give a system sufficient phase margin.

C. **Frequency response of lag compensator**:

1. The transfer function of a lag compensator with unity zero-frequency gain is of the form,

$$D(s) = \frac{\tau s + 1}{\beta \tau s + 1} \quad \beta > 1, \tau > 0$$

2. The sinusoidal transfer function is of the form,

$$D(j\omega) = \frac{1 + j\omega\tau}{1 + j\omega\beta\tau}$$

3. The bode plot of $D(j\omega)$, shown in Fig. 5.5.1, has two corner frequencies at $\omega = 1/\beta\tau$ and $\omega = 1/\tau$.

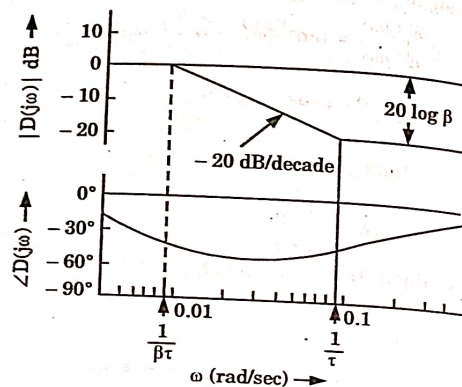


Fig. 5.5.1

4. The phase lag mainly occurs within and around the two corner frequencies.
5. It must be recognized here that any phase lag at the gain crossover frequency of the compensated system is undesirable.
6. To prevent detrimental effects of phase lag due to the lag compensator, the corner frequencies of the lag compensator must be located substantially lower than the gain crossover frequency of the compensated system.
7. Fig. 5.5.1 also shows that in the high frequency range, the lag compensator has an attenuation of $20 \log \beta$ dB, which is the property that is utilized to give a system sufficient phase margin.
8. The addition of a lag compensator results in an improvement in the ratio of control signal to noise signal in the loop.
9. The high frequency noise signals are attenuated by a factor $\beta > 1$, while the low frequency control signals undergo unit amplification (0 dB gain).
10. The choice of β is usually restricted because a very large β will appreciably reduce the gain crossover frequency and consequently speed of response of the system. A typical choice of β is 10.

Que 5.6. Explain Bode plot method to design a lag compensator.

OR

What are the effects of lag compensation?

Answer

Design procedure of lag compensator :

Step 1 : Let the system transfer function of controller be

$$G_c(s) = K_c \frac{s + \frac{1}{\tau}}{s + \frac{1}{\beta\tau}} \quad (\beta > 1)$$

$$G_c(s) = \beta K_c \frac{\tau s + 1}{\beta\tau s + 1}$$

$$K_c \beta = K$$

Let

The OLTF of the compensated system is

$$G_c(s)G(s) = K \frac{\tau s + 1}{\beta\tau s + 1} G(s) = \frac{\tau s + 1}{\beta\tau s + 1} KG(s)$$

$$= \frac{\tau s + 1}{\beta\tau s + 1} G_1(s) \quad \text{where } G_1(s) = KG(s)$$

Now, determine gain K to satisfy the requirement on the given static velocity error constant.

Step 2 : Using the value of K determined in step 1, draw Bode plot of $G_1(j\omega)$. Find the phase margin (ϕ).

Step 3 : Using specified phase margin (ϕ_2), find the required phase margin ϕ_1

$$\phi_1 = \phi_2 + \phi', \text{ where } \phi' = 5^\circ \text{ to } 12^\circ$$

Step 4 : Find the frequency at which the phase angle of the open loop transfer function is equal to -180° plus the required margin (ϕ_1). This is the new gain crossover frequency.

Step 5 : Determine attenuation necessary to bring the magnitude curve down to 0 dB at the new gain crossover frequency. This change is due to the factor (β) and the attenuation is $-20 \log \beta$. For this shift find the value of β .

Step 6 : Determine the other corner frequency (corresponding to pole of lag compensator) from $\omega_{c1} = 1/(\beta\tau)$.

Step 7 : Using the value of K determined in step 1 and β determined in step 5, calculate constant K_c as

$$K_c = \frac{K}{\beta}$$

Step 8 : Using the transfer function of lag compensator, draw Bode plot, and verify specifications.

- B. Effects of lag compensation :** It allows high gain at low frequencies thus it is basically a low pass filter. Hence it improves the steady state performance.

Que 5.7. Explain lead-lag compensator. Also write the effects of lead-lag compensation ?

Answer

A. Lead-lag compensator :

1. Lead-lag compensator is combination of lead network and lag network.

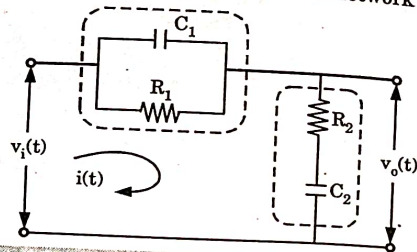


Fig. 5.7.1. Lead-lag compensator in time-domain.

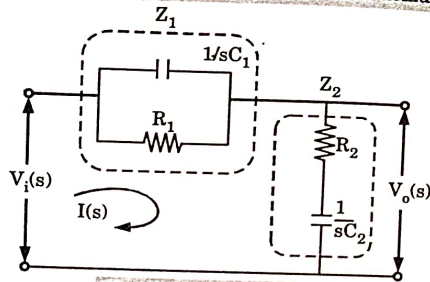


Fig. 5.7.2. In s-domain.

2. From Fig. 5.7.2, we have

$$Z_1 = R_1 \parallel \frac{1}{sC_1} = \frac{R_1 \times \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{sR_1C_1 + 1} \quad \dots(5.7.1)$$

and

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{(sR_2C_2 + 1)}{sC_2} \quad \dots(5.7.2)$$

3. Applying Kirchhoff's voltage law to the shown Fig. 5.7.1 network

$$V_i(s) = (Z_1 + Z_2) I(s) \quad \dots(5.7.3)$$

$$V_o(s) = Z_2 I(s) \quad \dots(5.7.4)$$

Hence the transfer function of the system

$$G_c(s) = \frac{V_o(s)}{V_i(s)} \quad \dots(5.7.5)$$

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4. Putting value of $V_o(s)$ and $V_i(s)$ in eq. (5.7.5), we have

$$G_c(s) = \frac{Z_2 I(s)}{(Z_1 + Z_2) I(s)}$$

$$G_c(s) = \frac{Z_2}{Z_1 + Z_2} \quad \dots(5.7.6)$$

5. Putting value of Z_1 and Z_2 in eq. (5.7.6), we have

$$G_c(s) = \frac{(sR_2C_2 + 1)}{sC_2} \times \frac{1}{\frac{R_1}{sR_1C_1 + 1} + R_2 + \frac{1}{sC_2}}$$

$$G_c(s) = \frac{(sR_2C_2 + 1)(sR_1C_1 + 1)}{s^2 R_1 R_2 C_1 C_2 + s R_2 C_2 + s R_1 C_1 + s R_1 C_2 + 1}$$

$$= \frac{\left(s + \frac{1}{R_2 C_2}\right) \left(s + \frac{1}{R_1 C_1}\right)}{s^2 + \left[\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1}\right] s + \frac{1}{R_1 C_1 R_2 C_2}}$$

$$G_c(s) = \frac{\left(s + \frac{1}{T_1}\right) \left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\beta}{T_1}\right) \left(s + \frac{1}{\beta T_2}\right)}$$

where,

$$T_1 = R_1 C_1, T_2 = R_2 C_2$$

$$\frac{\beta}{T_1} + \frac{1}{\beta T_2} = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1}$$

and

$$\alpha \beta T_1 T_2 = R_1 R_2 C_1 C_2$$

and

$$\alpha \beta = 1$$

6. Poles are at $s = -\frac{\beta}{T_1}, -\frac{1}{\beta T_2}$

7. Zeros are at $s = -\frac{1}{T_1}, -\frac{1}{T_2}$

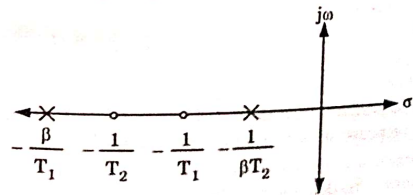


Fig. 5.7.3. Pole-zero configurations.

8. The various corner frequencies of the lag-lead compensator are,

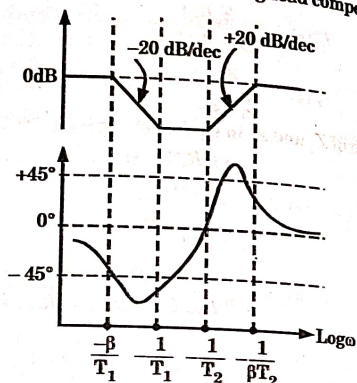


Fig. 5.7.4. Bode plot of lag-lead compensator.

$$\omega_{C1} = \frac{\beta}{T_1}, \text{ for a simple pole}$$

$$\omega_{C2} = \frac{1}{T_1}, \text{ for a simple zero}$$

$$\omega_{C3} = -\frac{1}{T_1}, \text{ for a simple zero}$$

$$\omega_{C4} = -\frac{1}{\beta T_2}, \text{ for a simple pole}$$

B. Effects:

1. Lag-lead compensator increases the low frequency gain which improves the steady state.
2. It increases bandwidth of the system which makes the system response very fast.

Que 5.8. Write down the procedure for designing lag-lead compensator.

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Answer

1. Evaluate the performance of the uncompensated system to determine how much improvement in transient response is required.
2. Design the lead compensator to meet the transient response specifications. The design includes the zero location, pole location, and the loop gain.

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3. Simulate the system to be sure all requirements have been met.
4. Redesign if the simulation shows that requirements have not been met.
5. Evaluate the steady state error performance for the lead-compensated system to determine how much more improvement in steady-state error is required.
6. Design the lag compensator to yield the required steady-state error.
7. Simulate the system to be sure all requirements have been met.
8. Redesign if the simulation shows that requirements have not been met.

PART-2

State Space Technique : The Concept of State and Space.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.9. Define state and state variable ? What are the advantages of state space techniques ?

OR

Define the following terms :

- i. State
- ii. State variables
- iii. State vector
- iv. State space
- v. State equation

Answer

- i. **State** : The state of a dynamic system is the smallest set of variables such that the knowledge of these variables at $t = t_0$ with the knowledge of the input for $t \geq t_0$ completely determines the behaviour of the system for any time $t \geq t_0$.
- ii. **State variables** : The variables involved in determining the state of dynamics system are called state variables.
- iii. **State vector** : If we need n variable to completely describe the behaviour of a given system, then these n state variables may be considered as n component of a vector x . Such a vector is called state vector.
- iv. **State space** : The n -dimensional space whose coordinate axes consists of the x_1 axis, x_2 axis ..., x_n axis is called state space. Any state can be represented by a point in the state space.

v. **State equation** : A state space representation is a mathematical model of the physical system as a set of output, input and state variables related by a differential equation is known as state equation.

Advantages of state space techniques :

1. The method takes into account the effect of all initial conditions.
2. It can be applied to non-linear as well as time varying conditions.
3. It can be conveniently applied to multiple input multiple output systems.
4. The system can be designed for the optimal conditions precisely by using this modern method.

PART-3

State Space Model of Physical System, Conversion of State Space to Transfer Function Model and Vice-Versa.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.10. Discuss state space representation.

Answer

1. In a state variable system, state variables are represented by $x_1(t)$, $x_2(t)$...
2. Input variables by $u_1(t)$, $u_2(t)$,
3. Output variables by $y_1(t)$, $y_2(t)$... where,

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix}; u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ \vdots \\ u_n(t) \end{bmatrix}; y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ \vdots \\ y_n(t) \end{bmatrix}$$

4. So, dynamics of a system can be represented by n^{th} order differential equation

$$\frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + a_2 \frac{d^{n-2} y(t)}{dt^{n-2}} \dots + a_{n-1} \frac{dy(t)}{dt} + a_n y(t) = u(t) \dots (5.10.1)$$

5. The knowledge of initial conditions $y(0)$, $\frac{dy(0)}{dt}$, ..., $\frac{d^{n-1} y(0)}{dt^{n-1}}$, along with the input $u(t)$ for $t \geq 0$, completely determines the future behavior of the system.

6. We take $y(t)$, $\frac{dy(t)}{dt}$, ..., $\frac{d^{n-1} y(t)}{dt^{n-1}}$ as a set of n -state variables.

7. Let us define

$$\begin{aligned} x_1 &= y \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \end{aligned}$$

8. Now from eq. (5.9.1)

$$\dot{x}_n = -a_n x_1 - a_{n-1} x_2 \dots - a_1 x_n + u$$

9. We can write this set of equations in matrix form as,

$$\dot{x} = Ax + Bu$$

Output :

$$y = Cx$$

where,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix};$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

10. For n^{th} order single-input-single-output system :

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

A = System matrix of order $(n \times n)$

B = Input coupling matrix of order $(n \times 1)$

C = Output coupling matrix of order $(1 \times n)$

x = State vector of order $(n \times 1)$

u = Scalar input of order (1×1)

y = Scalar output of order (1×1)

11. For multivariable system

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}; x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}; u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{bmatrix}$$

$$C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{p1} & c_{p2} & \dots & c_{pn} \end{bmatrix}$$

Que 5.11. Describe different representation of state model in details.

Answer

Basic ways to represent state models are :

- State space representation using differential equation :**
- The state model is represented using these state variables called as phase variables.
 - The phase variable is defined as those particular state variables which are obtained from one of the system variables and its derivatives.
 - Transfer function can be obtained by differential equation

$$\frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_{n-1} \frac{dy(t)}{dt} + a_n y(t) = b u(t)$$

$$\dot{y}^{(n)} + a_1 \dot{y}^{(n-1)} + \dots + a_{n-1} \dot{y}^{(1)} + a_n = b u(t)$$

Put

$$x_1 = y$$

$$x_2 = \dot{y} = \dot{x}_1$$

$$x_3 = \ddot{y} = \dot{x}_2$$

$$x_n = \dot{y}^{(n-1)} = \dot{x}_{n-1}$$

$$x_1 = y$$

$$\dot{x}_1 = x_2 = \dot{y}$$

$$\dot{x}_2 = x_3 = \dot{y}$$

$$\vdots$$

$$\vdots$$

$$\dot{x}_n = \dot{y}^{(n)} = bu - a_n - \dots - a_2 \dot{y}^{(n-2)} - a_1 \dot{y}^{(n-1)}$$

$$\dot{x}_n = bu - a_n x_3 - a_{n-1} x_2 - \dots - a_2 x_{n-1} - a_1 x_n$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b \end{bmatrix} u$$

$$\dot{x} = Ax + Bu$$

$$y = [1 \ 0 \ 0 \ \dots \ 0] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + [0]u$$

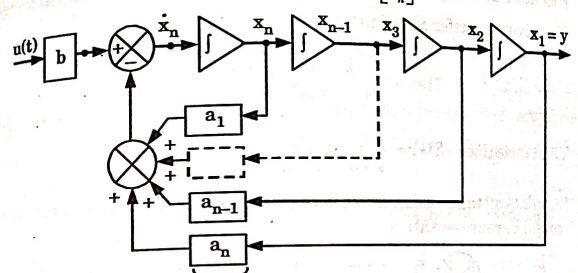


Fig. 5.11.1. The block diagram.

- Direct decomposition :** In direct decomposition we represent the denominator polynomial as

$$= \frac{1}{(s[s+a]+b)s+c}$$

We can do it for higher order also.

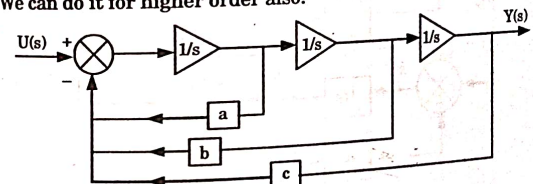


Fig. 5.11.2.

- Cascaded decomposition :** Here transfer function is written in terms of product of two or more small transfer functions.

$$\frac{Y(s)}{U(s)} = K \frac{(s+b_1)}{(s+a_1)} \times \frac{(s+b_2)}{(s+a_2)}$$

Each group is then decomposed by direct decomposition and the state diagram of the transfer functions is cascaded.

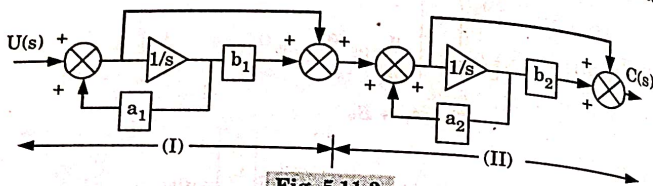


Fig. 5.11.3.

- d. **Parallel decomposition:** This method is also called canonical form. The given transfer function

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \dots + b_n}{a_0 s^n + a_1 s^{n-1} + \dots + a_n}$$

Then factorized $T(s) = \frac{c_1}{s+a_1} + \frac{c_2}{s+a_2} + \dots + \frac{c_n}{s+a_n}$

Simulate all terms using direct decomposition and add in parallel to have complete representation.

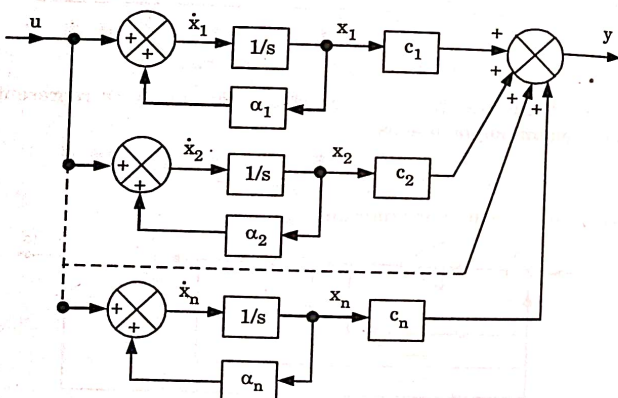


Fig. 5.11.4.

Que 5.12. Obtain the state space representation for the following differential equation $\ddot{y} + 5\dot{y} + 7y = 114u$, where 'y' is the output and 'u' is the input.

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Answer

In the given equation the input 'u' is not mentioned. Therefore assuming it as follows:

$$\ddot{y} + 5\dot{y} + 7y = 114u$$

1. Select state variables

$$y_1 = y$$

$$\dot{y}_1 = \dot{y} = y_2$$

$$\dot{y}_2 = 114u(t) - 5y_2 - 7y_1$$

2. State space representation:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -7 & -5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 114 \end{bmatrix} u(t)$$

Que 5.13. Construct the state model of a system characterized by the differential equation. Give the block diagram representation of the state model.

$$\frac{d^3 y}{dt^3} + 6\frac{d^2 y}{dt^2} + 11\frac{dy}{dt} + 6y = u$$

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Answer

$$1. \frac{d^3 y}{dt^3} + 6\frac{d^2 y}{dt^2} + 11\frac{dy}{dt} + 6y = u$$

2. Taking Laplace on both sides: $Y(s)(s^3 + 6s^2 + 11s + 6) = U(s)$

$$Y(s)/U(s) = 1/s^3 + 6s^2 + 11s + 6$$

3. Choosing state variable: $\dot{x}_1 = x_2, \dot{x}_2 = x_3$

$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 + u \text{ and } y = x_1$$

4. State model,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

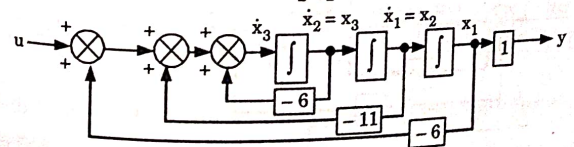


Fig. 5.13.1.

Que 5.14. Determine the state model from transfer function of a system given as

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 3s + 2}{s^3 + 9s^2 + 26s + 24}$$

Answer

$$1. \quad \frac{Y(s)}{X(s)} = (s^2 + 3s + 2)$$

Taking inverse Laplace both sides

$$\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t) = y(t)$$

$$2. \quad \frac{X(s)}{U(s)} = \frac{1}{s^3 + 9s^2 + 26s + 24}$$

Taking inverse Laplace both sides

$$\frac{d^3x(t)}{dt^3} + 9\frac{d^2x(t)}{dt^2} + 26\frac{dx(t)}{dt} + 24x(t) = u(t)$$

3. Choose state variables, $x(t) = x_1(t)$

$$\frac{dx(t)}{dt} = \dot{x}_1(t) = x_2(t)$$

$$\frac{d^2x(t)}{dt^2} = \dot{x}_2(t) = x_3(t)$$

$$\text{So,} \quad \begin{aligned} \dot{x}_3(t) &= 4(t) - 9x_3(t) - 26x_2(t) - 24x_1(t) \\ y(t) &= 2x_1(t) + 3x_2(t) + x_3(t) \end{aligned}$$

4. In state space representation form

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [2 \quad 3 \quad 1] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

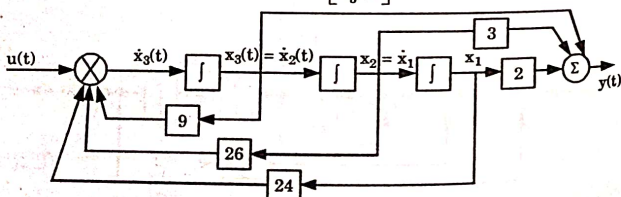


Fig. 5.14.1. Block diagram.

Que 5.15. Explain the correlation between transfer function and state space equations.

OR

What is transfer function? Also derive the expression for transfer function of a state model.

Answer

Transfer Function: Transfer function is defined as the ratio of Laplace transform of output of the system to the Laplace transform of input, under the assumptions that all initial conditions are zero.

$$T(s) = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}} = \frac{C(s)}{R(s)}$$

B. Derivation :

1. Let us consider a vector matrix differential equation

$$\dot{x} = Ax + Bu$$

and output, $y = Cx$

2. Now, taking Laplace transform with zero initial conditions

$$sX(s) = AX(s) + BU(s)$$

$$X(s) = [sI - A]^{-1} BU(s)$$

and

$$Y(s) = CX(s)$$

$$Y(s) = C [sI - A]^{-1} BU(s)$$

3. For a single-input-single-output system, Y and U are scalars.

4. Now transfer matrix can be given as

$$\text{Transfer matrix} = \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B$$

$$5. \quad \text{Transfer function} = C[sI - A]^{-1} B = \frac{C \text{adj}([sI - A])B}{\det [sI - A]}$$

6. Denominator part i.e. $|sI - A|$ is called the characteristic equation.

$$|sI - A| = 0$$

7. n^{th} degree characteristic equation $|sI - A| = 0$ has n roots or eigen values.

Que 5.16. Explain the term : State, State Space, State Vector. A SISO system has transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^3 + 7s^2 + 14s + 8}$$

write down the state equation and stage diagram.

AKTU 2019-20, Marks 07

Answer

A. State, state space and state vector : Refer Q. 5.9, Page 5-16C, Unit-5.

B. Numerical :

1. The given transfer function can be written as,

$$\frac{Y(s)}{X_1(s)} \cdot \frac{X_1(s)}{U(s)} = 1 \times \frac{1}{s^3 + 7s^2 + 14s + 8}$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3 + 7s^2 + 14s + 8}$$

2. The differential equation in time domain is written as,

$$\ddot{x}_1 + 7\dot{x}_1 + 14x_1 + 8x(t) = u(t)$$

Let

$$\dot{x}_1 = x_2 = 0.x_1 + 1.x_2 + 0.x_3 + 0.u$$

$$\dot{x}_2 = x_3 = 0.x_1 + 0.x_2 + 1.x_3 + 0.u$$

$$\dot{x}_3 = -8x_1 - 14x_2 - 7x_3 + 1.u$$

3. In matrix form,

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

As

$$\frac{Y(s)}{X_1(s)} = 1$$

4. It can be written in time domain as

$$y(t) = x_1(t) = 1.x_1 + 0.x_2 + 0.x_3$$

5. In matrix form $y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

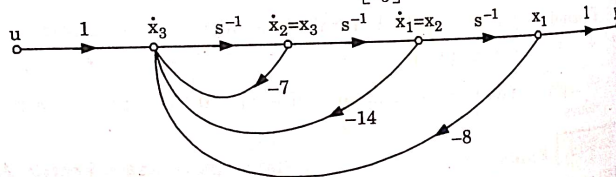


Fig. 5.16.1.

Que 5.17. For the following state equation, determine the transfer function $Y(s)/U(s)$,

$$\dot{x}(t) = \begin{bmatrix} 0 & 3 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 3 \end{bmatrix} u(t)$$

$$y(t) = [1 \ 0] x(t) + u(t)$$

Answer

$$\frac{Y(s)}{U(s)} = C[sI - A]^{-1} B + D$$

1.

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -2 & -3 \end{bmatrix}$$

2.

$$= \begin{bmatrix} s & -3 \\ 2 & s+3 \end{bmatrix}$$

3.

$$[sI - A]^{-1} = \begin{bmatrix} \frac{s+3}{s(s+3)+6} & \frac{3}{s(s+3)+6} \\ \frac{-2}{s(s+3)+6} & \frac{s}{s(s+3)+6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+3}{s^2+3s+6} & \frac{3}{s^2+3s+6} \\ \frac{-2}{s^2+3s+6} & \frac{s}{s^2+3s+6} \end{bmatrix}$$

4.

$$C[sI - A]^{-1} B = [1 \ 0] \begin{bmatrix} \frac{s+3}{s^2+3s+6} & \frac{3}{s^2+3s+6} \\ \frac{-2}{s^2+3s+6} & \frac{s}{s^2+3s+6} \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$= [1 \ 0] \begin{bmatrix} \frac{9}{s^2+3s+6} \\ \frac{3s}{s^2+3s+6} \end{bmatrix} = \frac{9}{s^2+3s+6}$$

5.

$$\frac{Y(s)}{U(s)} = \frac{9}{s^2+3s+6} + 1 = \frac{9+s^2+3s+6}{s^2+3s+6}$$

$$\frac{Y(s)}{U(s)} = \frac{s^2+3s+15}{s^2+3s+6}$$

PART-4

State Transition Matrix.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.18. What is the state transition matrix? Derive its expression. Enlist its properties with proofs.

Answer

A. State transition matrix (STM): The matrix $\phi(t) = \exp(At)$ is an $n \times n$ matrix and it helps in transition from initial state $X(0)$ to any other state $x(t)$ for $t > 0$, hence $\phi(t)$ is called state transition matrix.

B. Derivation :

1. Consider homogeneous equation, with $u(t) = 0$

$$\dot{x} = Ax$$

Laplace transform,

$$sX(s) - x(0) = A X(s)$$

$$(sI - A) X(s) = x(0)$$

I = Identity matrix

s = Scalar Laplace operator.

2. Multiplying both sides by $[sI - A]^{-1}$

$$X(s) = [sI - A]^{-1} x(0)$$

$$X(s) = \phi(s) x(0)$$

where,

$$\phi(s) = [sI - A]^{-1}$$

$$\phi(s) = \frac{1}{s} \left[I - \frac{A}{s} \right]^{-1} = \frac{I}{s} + \frac{A}{s^2} + \frac{A^2}{s^3} + \dots$$

3. Taking inverse Laplace transform of $X(s)$,

$$x(t) = L^{-1} [X(s)] = L^{-1} [sI - A]^{-1} x(0)$$

$$= L^{-1} \left[\frac{I}{s} + \frac{A}{s^2} + \frac{A^2}{s^3} + \dots \right] x(0)$$

$$= \left[I + At + \frac{A^2 t^2}{2} + \dots \right] x(0)$$

$$x(t) = e^{At} x(0) = \phi(t) x(0)$$

4. State transition matrix

$$\phi(t) = L^{-1} [sI - A]^{-1} = L^{-1} [\phi(s)]$$

5. So, state transition matrix (STM) can be given as,

$$\phi(t) = e^{At} = I + At + \frac{1}{2} A^2 t^2 + \frac{1}{3} A^3 t^3 + \dots$$

5-28 C (EN-Sem-5)

C. Properties of STM:

$$\phi(0) = e^{A \cdot 0} = I$$

$$\phi(0) = e^{A \cdot 0} = I$$

$$\phi^{-1}(t) = \phi(-t)$$

$$\phi^{-1}(t) = \frac{1}{\phi(t)} = \frac{1}{e^{At}} = e^{-At} = \phi(-t)$$

Proof:

$$\phi(t_2 - t_1) \phi(t_1 - t_0) = \phi(t_2 - t_0)$$

Proof:

$$\phi(t_2 - t_1) \phi(t_1 - t_0) = e^{A(t_2 - t_1)} e^{A(t_1 - t_0)} = e^{A(t_2 - t_1 + t_1 - t_0)} = e^{A(t_2 - t_0)} = \phi(t_2 - t_0)$$

Proof:

$$[\phi(t)]^k = \phi(kt)$$

$$\text{Proof: } \phi(t)^k = \phi(t) \cdot \phi(t) \dots k \text{ times} = e^{At} e^{At} \dots k \text{ times} = e^{Akt} = \phi(kt)$$

Proof:

$$\phi(t_1 + t_2) = \phi(t_1) \phi(t_2) = \phi(t_2) \phi(t_1)$$

Proof:

$$\phi(t_1 + t_2) = e^{A(t_1 + t_2)} = e^{At_1} e^{At_2} = e^{At_2} e^{At_1} = \phi(t_2) \phi(t_1)$$

Proof:

$$\phi(t_1 + t_2) = e^{A(t_1 + t_2)} = e^{At_1} e^{At_2} = e^{At_2} e^{At_1} = \phi(t_2) \phi(t_1)$$

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Proof:

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Que 5.20. State properties of State Transition Matrix (STM), find out state transition matrix for

$$A = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}$$

AKTU 2021-22, Marks 10

Answer

A. Properties of state transition matrix : Refer Q. 5.18, Page 5-27C, Unit-5.

B. Numerical :

Given : $A = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}$

To Find : $\phi(t)$

1. $[sI - A] = s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 8 & s+6 \end{bmatrix}$
2. $\phi(s) = [sI - A]^{-1} = \frac{\text{Adj}[sI - A]}{|sI - A|} = \frac{\begin{bmatrix} s+6 & 1 \\ -8 & s \end{bmatrix}}{(s+2)(s+4)}$

$$= \begin{bmatrix} \frac{s+6}{(s+2)(s+4)} & \frac{1}{(s+2)(s+4)} \\ \frac{-8}{(s+2)(s+4)} & \frac{s}{(s+2)(s+4)} \end{bmatrix}$$
3. $\phi(t) = L^{-1} [sI - A]^{-1} = L^{-1} \begin{bmatrix} \frac{s+6}{(s+2)(s+4)} & \frac{1}{(s+2)(s+4)} \\ \frac{-8}{(s+2)(s+4)} & \frac{s}{(s+2)(s+4)} \end{bmatrix}$

$$= \begin{bmatrix} 2e^{-2t} - e^{-4t} & \frac{1}{2} [e^{-2t} - e^{-4t}] \\ 4(-e^{-2t} + e^{-4t}) & e^{-2t} + 2e^{-4t} \end{bmatrix}$$

PART-5

Concept of Controllability and Observability and Their Testing.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.21. State and explain controllability.

Answer

A. Controllability : A system is said to be controllable if the state of the system can be transferred to another desired state over a given time period by using input.

B. Kalman's test for controllability :

1. Consider n^{th} order multiple input linear time invariant system represented by its state equation as,

$$\dot{x} = Ax + Bu$$

2. The necessary and sufficient condition for the system to be completely state controllable is that the rank of the composite matrix Q_c is n .
3. The composite matrix Q_c is given by,

$$Q_c = [B : AB : A^2B : \dots : A^{n-1}B]$$

C. Gilbert's test for controllability :

1. For the Gilbert's test it is necessary that the matrix A must be in canonical form. Hence the given state model is required to be transformed to the canonical form first, to apply the Gilbert's test.
2. Consider single input linear time invariant system represented by,

$$\dot{z} = Az + Bu$$

where A is not in the canonical form. Then it can be transformed to the canonical form by the transformation,

$$x = Mz$$

where M = Model matrix

3. The transformed state model,

$$\dot{z} = \tilde{A}z + \tilde{B}u$$

where

$$\tilde{A} = M^{-1}AM$$

$$\tilde{B} = M^{-1}B$$

4. In such a case the necessary and sufficient condition for the complete state controllability is that the vector matrix $\tilde{B}$ should not have any zero elements. If it has zero elements then the corresponding state variables are not controllable.
5. If the eigen values are repeated then matrix A cannot be transformed to Jordan canonical form. If A has eigen values $\lambda_1, \lambda_1, \lambda_2, \lambda_2, \lambda_3, \lambda_4, \dots, \lambda_n$ then the transformation results Jordan canonical form shown in matrix below.
6. In such a case, the condition for the complete state controllability is that the elements of any row of $\tilde{B}$ that corresponds to the last row of each Jordan block are not all zero.

$$J = \begin{bmatrix} \boxed{\lambda_1 & 1} & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \boxed{\lambda_2 & 1} & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \lambda_2 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 1 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & \lambda_n \end{bmatrix}$$

Jordan block

Que 5.22. Explain the concept of observability.

Answer

A. Observability: A system is said to be observable if all the state of the system can be determined on the basis of knowledge of the output of the system at given time.

B. Kalman's test for observability:

1. State equation

$$\dot{x} = Ax + Bu$$

and

$$y = Cx$$

2. The system is completely observable if and only if the rank of the composite matrix Q_o is n .

Composite matrix Q_o is given by,

$$Q_o = [C^T : A^T C^T : \dots : (A^T)^{n-1} C^T]$$

where

C^T = Transpose of matrix C

A^T = Transpose of matrix A

C. Gilbert's test for observability:

1. For Gilbert's test, the state model must be expressed in the canonical form. Consider the state model of linear time invariant system as,

$$\dot{x} = Ax + Bu$$

and

$$y = Cx$$

2. Use the transformation $x = Mz$ where M is the model matrix.

$$y = CMz = \tilde{C}z$$

$\therefore$

$$\tilde{C} = CM$$

where

For a single input single output system,

$$y = \tilde{C}z = [\tilde{C}_{11} \ \tilde{C}_{12} \ \dots \ \tilde{C}_{1n}] \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$$

$$= \tilde{C}_{11}z_1 + \tilde{C}_{12}z_2 + \dots + \tilde{C}_{1n}z_n$$

3. For the system to be observable, each term corresponding to each state must be observed in the output. Hence none of the coefficient of $\tilde{C}$ must be zero.

Que 5.23. A single input single output system is given as

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } C = [1 \ 0 \ 0]$$

Test for controllability and observability.

AKTU 2018-19, Marks 07

Answer

$$\text{Given: } A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } C = [1 \ 0 \ 0]$$

To Test: Controllability and Observability.

A. Controllability test:

$$1. AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -3 & -3 \end{bmatrix}$$

$$2. A^2B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -3 \\ 11 & 11 \end{bmatrix}$$

3. Controllability test matrix is given by

$$Q_c = [B : AB : A^2B]$$

$$Q_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -3 & -3 \\ 1 & 1 & -3 & -3 & 11 & 11 \end{bmatrix}$$

$$4. \text{ Consider } \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -3 \end{bmatrix} = 1 \neq 0; |Q_c| \neq 0$$

5. Hence rank of Q_c is equal to its order i.e., 3. Therefore the system is controllable.
- B. Observability test :

$$1. \quad C = [1 \ 0 \ 0]; \quad C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$2. \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}; \quad A^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

$$3. \quad A^T C^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$4. \quad (A^T)^2 C^T = (A^T) (A^T C^T) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

5. Observability test matrix is given by

$$Q_o = [C^T : A^T C^T : (A^T)^2 C^T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|Q_o| = 1 \neq 0$$

6. Hence its rank is 3 equal to its order i.e., 3, therefore system is completely observable.

Que 5.24. The state equations are given below; check controllability and observability of a system.

$$\dot{x}_1 = x_2 - x_3 + 3r$$

$$\dot{x}_2 = x_1 + x_2 + x_3 - 2r$$

$$\dot{x}_3 = x_1 + x_2 + r$$

$$y = x_1$$

AKTU 2019-20, Marks 07

Answer

$$\text{Given : } A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, C = [1 \ 0 \ 0]$$

To Find : Controllability and observability.

Introduction to Design

5-33 C (EN-Sem-5)

A. Controllability :

$$AB = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

$$A^2 B = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 3 & 0 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

- 1.
2. Controllability test matrix is given by

$$Q_C = [B : AB : A^2 B] = \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}, |Q_C| \neq 0$$

3. Hence rank of Q_C is equal to its order i.e., 3 therefore the system is controllable.

B. Observability :

$$A^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}, C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$(A^T)^2 C^T = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- 1.
- 2.
3. Observability test matrix is given by

$$Q_o = [C^T : A^T C^T : (A^T)^2 C^T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = 1, |Q_o| \neq 0$$

4. Hence rank of Q_o is equal to its order i.e., 3, so the system is observable.

Que 5.25. Find the controllability and observability of the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0[u]$$

AKTU 2020-21, Marks 10

Answer

Given : $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C = [1 \ 0 \ 0]$

To Find : Controllability and observability.

A. Controllability test :

1. $AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

2. $A^2B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

3. Controllability test is given by

$$Q_C = [B : AB : A^2B]$$

$$|Q_C| \neq 0$$

4. Hence the rank of Q_C is equal to its order i.e., 3. Therefore the system is completely controllable.

B. Observability test :

1. $A^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

2. $A^T C^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

3. $(A^T)^2 C^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

4. Observability test matrix is given by,

$$Q_O = [C^T : A^T C^T : (A^T)^2 C^T]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_O \neq 0$$

6. Hence the rank of the matrix Q_O is equal to its order i.e., $n=3$. So the system is observable.

1
UNITControl System
Concepts
(2 Marks Questions)

1.1. What are the major types of control systems ? Explain them in detail with examples.

Ans. There are two types of control system :

1. Open loop control system :

In open loop control systems the control action is independent of the desired output. In this system the output is not compared with the reference input.

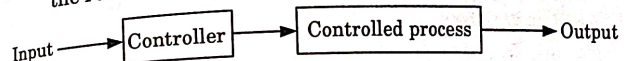


Fig. 1.1.1

Example : Immersion rod, Traffic control system.

2. Closed loop control system :

In a closed loop control system the output has an effect on control action through a feedback as shown in Fig. 1.1.2.

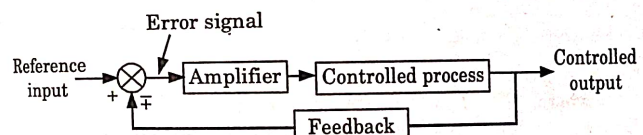


Fig. 1.1.2.

Example : Automatic steering control system, Electric iron.

1.2. List the advantages of closed loop over open loop system.

AKTU 2020-21, Marks 02

Ans.

1. Feedback reduces the sensitivity to parameter variation and noise.
2. It produces beneficial effect on bandwidth, impedance, frequency response.

1.3. Why negative feedback is preferred in closed loop control system ?

AKTU 2020-21, Marks 02

Ans. The negative feedback results in better stability in steady state and rejects any disturbance signals. It has also low sensitivity to parameter variations. Hence negative feedback is preferred in closed loop systems.

1.4. What are the applications of AC servomotor ?

Ans.

1. In feedback control systems
2. In aircraft and spacecraft
3. In tracking and guidance systems
4. In robotics.

1.5. What is a signal flow graph ?

Ans. A signal flow graph is a pictorial representation of the simultaneous equations describing the system. This is an alternate approach which does not require any reduction process because of availability of a flow graph gain formula which relates the input and output system variables.

1.6. What is Mason's gain formula ?

AKTU 2018-19, Marks 02

OR

Write the Mason's gain formula.

AKTU 2020-21, Marks 02

Ans. The overall gain can be determined by Mason's gain formula given below :

$$T = \sum_{k=1}^n \frac{P_k \Delta_k}{\Delta}$$

where,

P_k = The forward path gain

Δ = The graph determinant which involves closed-loop gain and mutual interactions between non-touching loops.

Δ_k = The path factor associated with the concerned path and involves all closed loops in the graph which are isolated from the forward path under consideration.

1.7. What are the advantages of negative feedback control system ?

Ans.

1. Better frequency response.
2. Less distortion.
3. Less gain or voltage drift.
4. Less temperature drift.

1.8. Draw the block diagram, which represent a driver driving a car.

AKTU 2021-22, Marks 02

Ans.

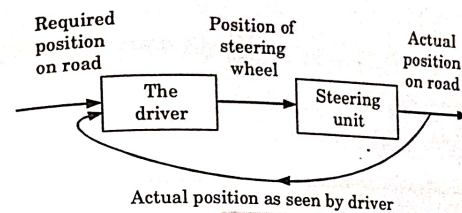


Fig. 1.8.1.

1.9. Define : Self loop and non-touching loop in signal flow graph by suitable example.

AKTU 2021-22, Marks 02

Ans.

1. **Self loop :** A feedback loop consisting of only one node is called self loop. i.e., G_{33} at x_3 is self loop.

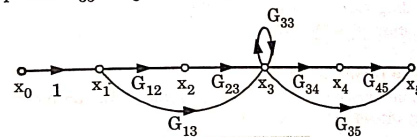


Fig. 1.9.1.

2. **Non-touching loops :** If there is no node common in between the two or more loops, such loops are said to be non-touching loops.

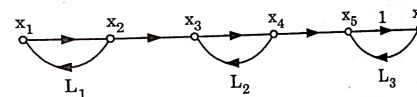


Fig. 1.9.2. Three non-touching loops.

1.10. Define transfer function. Give an example for it.

AKTU 2020-21, Marks 02

Ans.

- A. Transfer function :** It is defined as the Laplace transform of output to the Laplace transform of input when all initial conditions are set to zero.

$$T(s) = \frac{L[c(t)]}{L[r(t)]} = \frac{C(s)}{R(s)}$$

B. Example :

1. Find the transfer function $\frac{C(s)}{R(s)}$ of a system having differential equation given below,

$$\frac{2d^2c(t)}{dt^2} + 2\frac{dc(t)}{dt} + c(t) = r(t)$$

2. Taking laplace transform of eq. (1), we get
 $2s^2C(s) + 2sC(s) + C(s) = R(s)$
 $(2s^2 + 2s + 1)C(s) = R(s)$... (1)

$$\frac{C(s)}{R(s)} = \frac{1}{2s^2 + 2s + 1}$$

1.11. What are the applications of stepper motor ?**Ans.**

1. Paper feed motors in typewriters and printer
2. Recording heads in computer disk drives.
3. Positioning of print heads

1.12. What is tachometer ?**Ans.**

Tachometer is electromechanical device that convert mechanical energy into electrical energy.



Time Response Analysis (2 Marks Questions)

- 2.1. What are the standard test signals used in control system ?

AKTU 2020-21, Marks 02

Ans.

1. Step signal
2. Ramp signal
3. Parabolic signal
4. Impulse signal.

- 2.2. What is an impulse response ? AKTU 2018-19, Marks 02

Ans.

Impulse response is the response of system when we provide impulse signal as input.

- 2.3. What do you mean by settling time, write expression for 2nd order system ? AKTU 2021-22, Marks 02

Ans. Settling time is the time required for the response curve to reach and stay within a specified percentage (2 % to 5 %) of its final value.

$$\text{For 2 \% band, } t_s = \frac{4}{\xi\omega_n}$$

$$\text{and for 5 \% band, } t_s = \frac{3}{\xi\omega_n}$$

- 2.4. Define rise time for second order control system.

Ans.

Rise Time (t_r) : Rise time is defined as the time required for the response to rise from 10 % to 90 % (for overdamped) or from 0 % to 100 % (for underdamped) of its final value.

$$\text{Rise time } (t_r) = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_d}$$

25. Define damping ratio.

AKTU 2018-19, Marks 02

Ans. Damping ratio indicates the amount of damping present in system and denoted by ξ .

26. What is steady state error?

AKTU 2018-19, Marks 02

Ans. It indicates the error between the actual output and desired output as t tends to infinity i.e.

$$e_{ss} = \lim_{t \rightarrow \infty} [r(t) - c(t)]$$

It is denoted by e_{ss} .

27. Distinguish between type and order of a system.

AKTU 2020-21, Marks 02

Ans.

Type	Order
The number of poles at the origin gives the type of a transfer function.	The total number of poles gives the order of the transfer function.

28. Give example of type zero, type one and type two systems.

AKTU 2019-20, Marks 02

Ans. Type zero system :

$$G(s)H(s) = \frac{k(s+z_1)(s+z_2)}{s^0(s+p_1)(s+p_2)}$$

Type one system :

$$G(s)H(s) = \frac{k(s+z_1)(s+z_2)}{s^1(s+p_1)(s+p_2)}$$

Type two system :

$$G(s)H(s) = \frac{k(s+z_1)(s+z_2)}{s^2(s+p_1)(s+p_2)}$$

29. The OLTF of a unity feedback system is $G(s) = 1/s(s+1)(s+4)$ find the steady state error (ess) due to a unit step.

AKTU 2021-22, Marks 02

Ans. For unit step input steady state error,

$$e_{ss} = \frac{1}{1+K_p}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s) = \lim_{s \rightarrow 0} \frac{1}{s(s+1)(s+4)} \times 1 = \infty$$

$$e_{ss} = \frac{1}{1+\infty} = 0$$

2.10. What is damping constant give its relation with time constant?

AKTU 2019-20, Marks 02

Ans. The damping constant is a measure of a system's settling time. Relation with time constant : The damping constant is an inverse of time constant.

2.11. Define proportional controller.

Ans. It is a control system technology based on a response in proportion to the difference between what is set as a desired process variable (or set point) and the current value of the variable.

2.12. The impulse response of unity feedback control system is $c(t) = te^{-t} + 2be^{-t}$, find transfer function.

AKTU 2019-20, Marks 02

Ans.

Given : $c(t) = te^{-t} + 2be^{-t}$, $H(s) = 1$, $r(t) = \delta(t)$
To Find : $G(s)$.

1. Laplace transform of $r(t)$ and $c(t)$, $R(s) = 1$

$$\text{and } C(s) = \frac{1}{(s+1)^2} + \frac{2b}{s+1} = \frac{1+2b(s+1)}{(s+1)^2}$$

2. For unity feedback control system

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$C(s) = \frac{G(s)}{1+G(s)} \times R(s) \quad \dots(1)$$

3. Putting value of $C(s)$ and $R(s)$ in eq. (1), we get

$$\frac{1+2b(s+1)}{(s+1)^2} = \frac{G(s)}{1+G(s)} \times 1$$

$$G(s) = \frac{2bs+2b+1}{s^2+2s-2bs-2b+1}$$



3

UNIT

Stability and Algebraic Criteria (2 Marks Questions)

3.1. Define a stable system.

AKTU 2020-21, Marks 02

Ans:

1. A system is stable if its response (i.e., the transfer function) approaches zero as time approaches infinity.
2. In other words, a system is stable if every bounded input produces a bounded output.

3.2. State the necessary condition of stability.

AKTU 2020-21, Marks 02

Ans:

1. All the coefficients of characteristics equation must have same sign.
2. There should be no missing term.

3.3. State absolute stability and relative stability.

AKTU 2021-22, Marks 02

Ans:

1. **Absolute stability:** If a system output is stable for all variations of its parameters, then the system is called absolutely stable system. It gives the information about whether system is stable or unstable.
2. **Relative stability:** The system is said to be relatively more stable or unstable on the basis of settling time. System is said to be relatively more stable if settling time for the system is less than that of the other system. Relative stability gives degree of stability or how close it to instability.

3.4. What are the limitations of Routh Hurwitz criterion?

AKTU 2021-22, Marks 02

Ans:

1. It is valid only if the characteristics equation is algebraic.
2. If any coefficient of the characteristics equation is complex or contain power of e then this criterion cannot be applied

3.5. What is centroid in root locus? **AKTU 2018-19, Marks 02**

OR

What is centroid of asymptotes and how the centroid is calculated?

AKTU 2019-20, Marks 02

Ans: All asymptotes intersect the real axis at a common point known as centroid.

$$\sigma = \frac{\sum \text{Real part of open loop poles} - \sum \text{Real part of open loop zeros}}{P - Z}$$

where,

 P = Number of open loop poles Z = Number of open loop zeros.

3.6. Define the term breakaway point.

Ans: Breakaway point is defined as the point at which root locus comes out of the real axis.

3.7. What is the effect of addition of pole on root locus?

Ans: The addition of a pole to the system open loop transfer function pushes the root locus to the right of original root locus, thereby reducing the system stability and making the system more oscillatory.

3.8. What is the effect of addition of zero on root locus?

Ans: The addition of a zero to the system pushes the root locus to the left by original root locus, thereby making the system more stable and less oscillatory.

3.9. What is root contour?

Ans: The locus of root of characteristics equation obtained by varying parameter of the system other than K from 0 to ∞ is called root contour of the system.

3.10. Define asymptote.

Ans: The line which touches the curve at infinity is known as asymptote. For $(P - Z)$ branches, angle of such asymptote is given by:

$$\theta_q = \frac{(2q + 1) \times 180^\circ}{P - Z}$$

where $q = 0, 1, 2, \dots, (P - Z - 1)$.3.11. How will you find the gain K at a point on root locus?Ans: The gain K at a point $s = s_a$ on root locus is given by,

$$K = \frac{\text{Product of length of vector from open loop poles to the point } s_a}{\text{Product of length of vector from open loop zeros to the point } s_a}$$


4

UNIT

Frequency Response Analysis (2 Marks Questions)

4.1. What is frequency response ?

Ans. The magnitude and phase relationship between the sinusoidal input and the steady state output of a system is termed as the frequency response.

4.2. What are the advantage and disadvantage of frequency response ?

Ans.

- A. **Advantage :** These methods are easy to use for design of control and for finding absolute as well as relative stability of the system.
B. **Disadvantage :** The frequency response test cannot be performed on non-interruptible systems.

4.3. Write the expression for resonant peak and resonant frequency.

Ans.

1. Resonant peak, $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$;
2. Resonant frequency, $\omega_r = \omega_n\sqrt{1-2\xi^2}$

4.4. Define gain and phase margins. **AKTU 2020-21, Marks 02**

Ans.

- A. **Gain margin :** It is the reciprocal of magnitude $|G(j\omega)|$ at the frequency at which the phase angle is -180° .

$$\text{Gain Margin (GM), } K_g = \frac{1}{|G(j\omega_c)|}$$

where, ω_c = Phase cross-over frequency.

- B. **Phase margin :** The phase margin is that amount of the additional phase lag at the gain crossover frequency required to bring the system to the verge of instability. Phase margin is equal to 180° plus the angle of $G(j\omega)$ at the gain crossover point, $\phi_m = 180^\circ + \phi$

4.5. Explain gain crossover frequency margin, phase crossover frequency. **AKTU 2019-20, Marks 02**

OR

Define gain crossover frequency and phase margin.

AKTU 2018-19, Marks 02

Ans.

- A. **Gain crossover frequency :** Gain crossover frequency is the frequency at which magnitude of open loop transfer function is unity.

- B. **Crossover frequency :** Phase crossover frequency is the frequency at which phase of open loop transfer function is -180° .

- C. **Phase margin :** Refer Q. 4.4, Page SQ-10C, Unit-4, 2 Marks Questions.

4.6. What are the advantage and disadvantage of polar plot ?

Ans.

- A. **Advantage :** It shows the frequency response characteristics of a system over the entire frequency range in a single plot.

- B. **Disadvantage :** It does not indicate the contribution of each individual factor of the open loop transfer function.

4.7. Sketch polar plot of $G(s) = 1/(1 + \alpha s)$.

AKTU 2019-20, Marks 02

Ans.

1. The frequency domain transfer function is,

$$G(j\omega)H(j\omega) = \frac{1}{1 + \alpha j\omega} = \frac{1 + j0}{1 + j\omega\alpha}, \quad |G(j\omega)H(j\omega)| = M = \frac{1}{\sqrt{1 + \omega^2 \alpha^2}}$$

$$\angle G(j\omega)H(j\omega) = \phi = \frac{\tan^{-1}\left(\frac{0}{1}\right)}{\tan^{-1}\left(\frac{\omega\alpha}{1}\right)} = \frac{0^\circ}{(\tan^{-1} \omega\alpha)} = -\tan^{-1}(\omega\alpha)$$

2. For various values of ω , the value of M and ϕ are :

ω	Magnitude (M)	Phase angle (ϕ)
0	1	0°
∞	0	-90°

4. The corresponding polar plot is shown in Fig. 4.7.1.

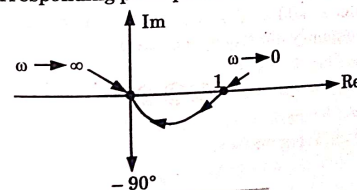


Fig. 4.7.1

4.8. Draw the polar plot $G(s) = 1/(s+2)$.

AKTU 2021-22, Marks 02

Ans:

1. Put $s = j\omega$, $G(j\omega) = \frac{1}{(j\omega + 2)}$

2. $|G(j\omega)| = M = \frac{1}{\sqrt{\omega^2 + 4}}$, $\angle G(j\omega) = \phi = -\tan^{-1} \left(\frac{\omega}{2} \right)$

3. For various values of ω , the value of M and ϕ are :

ω	Magnitude (M)	Phase angle (ϕ)
0	0.5	0°
∞	0	-90°

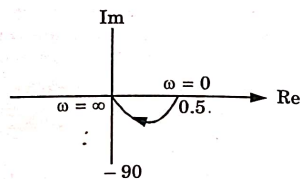


Fig. 4.8.1

4.9. Write the condition to test the stability of Nyquist plot.

Ans: Condition to test stability, $N = Z - P$.where, N = Number of encirclement of $(-1 + j0)$ point in clockwise direction. Z = Number of closed loop poles on the right half of s -plane.For system to be stable, Z should be zero i.e., $N = -P$.

4.10. Write advantages of Bode Plot. AKTU 2021-22, Marks 02

Ans:

1. The magnitudes are expressed in dB, and so, a simple procedure is available to add magnitude to each one by one.
2. The frequency domain specifications can be easily determined.



Que 5.26. Determine the state controllability and observability of the system described

$$\dot{x} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 3 \\ -7 & 5 & 9 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \text{ and } y = [5 \ 2 \ 7]x$$

AKTU 2021-22, Marks 10

Answer

The procedure is same as Q. 5.23, Page 5-32C, Unit-5.

1. **Controllability test** : Hence rank of Q_c is equal to order i.e., 3. Therefore the system is controllable.
2. **Observability test** : Hence its rank of Q_o is equal to 3, therefore the system is completely observable.

Que 5.27. Discuss about the concept of SISO and MIMO.

Answer

SISO (Single Input Single Output) and MIMO (Multiple Input Multiple Output) are key concepts in control systems, representing the number of inputs and outputs that a system has.

A. **SISO (Single Input Single Output) :**

1. A SISO system has one input and one output.
2. SISO systems are straightforward to analyze because the relationship between the input and output is direct, typically described by a transfer function or state-space model.
3. Since there's only one input influencing one output, the control design is simpler.

4. **Example** : A temperature control system.B. **MIMO (Multiple Input Multiple Output) :**

1. A MIMO system has multiple inputs and multiple outputs.
2. In MIMO systems, each input can affect multiple outputs, and each output may depend on more than one input.
3. This interaction requires more sophisticated control strategies, often involving matrix mathematics or state-space analysis.
4. **Example** : Aircraft control.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Define Phase lead compensation technique. How would you draw its pole zero configuration ?

Ans. Refer Q. 5.1, Unit-5.

Q. 2. Design a lead compensator for a system whose open loop transfer function is

$$G(s)H(s) = 4/s(s+2)$$

It will fulfill following requirement

- Static velocity error constant = 20 sec^{-1}
- PM at least 50°
- GM at least 10 db

Ans. Refer Q. 5.3, Unit-5.

Q. 3. Explain the lag compensation.

Ans. Refer Q. 5.4, Unit-5.

Q. 4. Explain lead-lag compensator. Also write the effects of lead-lag compensation ?

Ans. Refer Q. 5.7, Unit-5.

Q. 5. Describe different representation of state model in details.

Ans. Refer Q. 5.11, Unit-5.

Q. 6. Construct the state model of a system characterized by the differential equation. Give the block diagram representation of the state model.

$$\frac{d^3 y}{dt^3} + 6 \frac{d^2 y}{dt^2} + 11 \frac{dy}{dt} + 6y = u$$

Ans. Refer Q. 5.13, Unit-5.

Q. 7. Explain the correlation between transfer function and state space equations.

Ans. Refer Q. 5.15, Unit-5.



Introduction to Design (2 Marks Questions)

5.1. What is compensation ?

Ans. The compensation is the design procedure in which the system behaviour is altered to meet the desired specifications, by introducing additional device called compensator.

5.2. Why compensators are used in control system, what is effect of lag compensator ?

AKTU 2021-22, Marks 02

Ans.

A. Reason :

1. Compensate an unstable system to make it stable.
2. A compensating network is used to minimize overshoot.
3. These compensating networks increase the steady state accuracy of the system.

B. Effect of lag compensators : It allows high gain at low frequencies thus it is basically a low pass filter. Hence it improves the steady state performance.

5.3. What are the effects of lead compensation ?

Ans.

1. The lead compensator adds a dominant zero and a pole. This increases the damping of the closed loop system.
2. It improves the phase margin of the closed loop system.
3. It increases bandwidth of the closed loop system.

5.4. Define state and state variable. AKTU 2020-21, Marks 02

Ans.

A. State : State of a system is the minimum amount of information needed along with initial conditions at $t = t_0$ and input excitation so that future response of system can be completely described at any time $t > t_0$.

B. State Variables : A set of at least n variables $x_1(t), x_2(t) \dots x_n(t)$, are needed to completely describe how a system will behave in future, along with initial state and input excitation. These minimal set of variables which can determine the state of a system are known as state variables.

5.5. Define state variable and state space.

AKTU 2018-19, Marks 02

- Ans:**
1. **State variables:** The variables involved in determining the state of dynamics system are called state variables.
 2. **State space:** The n -dimensional space whose coordinate axes consists of the x_1 axis, x_2 axis ..., x_n axis is called state space. Any state can be represented by a point in the state space.

5.6. Discuss the advantages of state variable technique over transfer function approach.

- Ans:**
1. This method can be applied to linear or non linear, time variant or time invariant system.
 2. This method can be designed for optimal conditions.

5.7. What is state transition matrix?

- Ans:** The matrix that satisfies the linear homogeneous state equation is known as state transition matrix.

$$\phi(t) = e^{At} = 1 + At + \frac{A^2 t^2}{2!} + \frac{A^3 t^3}{3!} \dots$$

5.8. Enlist the properties of state transition matrix.

AKTU 2019-20, Marks 02

Ans:

1. $\phi(0) = I$, I = Identity matrix
2. $\phi^{-1}(t) = \phi(-t)$
3. $[\phi(t)]^k = \phi(kt)$

5.9. Explain the concept of controllability and observability.

Ans:

1. **Controllability:** A system is said to be controllable if the state of the system can be transferred to another desired state over a given time period by using input.
2. **Observability:** A system is said to be observable if all the state of the system can be determined on the basis of knowledge of the output of the system at given time.

5.10. Find the eigen vectors of the matrix

$$A = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$$

AKTU 2021-22, Marks 02

Ans:

1. The characteristics equation for eigen values, $|(A - \lambda I)| = 0$

$$\begin{vmatrix} -3-\lambda & 1 \\ 1 & -3-\lambda \end{vmatrix} = 0$$

$$(-3-\lambda)(-3-\lambda) - 1 = 0$$

$$\lambda^2 + 6\lambda + 8 = 0$$

$$\lambda^2 + 4\lambda + 2\lambda + 8 = 0$$

$$(\lambda + 2)(\lambda + 4) = 0$$

2. Solving characteristic equation,

$$\lambda = -2, \lambda_2 = -4$$

These are the eigen values of matrix A.

3. To find eigen vectors, obtain matrix $[A - \lambda I]$ for each eigen value of λ in eq. (1).

4. For $\lambda_1 = -2$, $[A - \lambda I] = \begin{bmatrix} -5 & 1 \\ 1 & -5 \end{bmatrix}$

$$M_1 = \begin{bmatrix} C_{11} \\ C_{12} \end{bmatrix}$$

where, C_{11}, C_{12} co-factors of row 1

$$M_1 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$$

5. For $\lambda_2 = -4$, $[A - \lambda I] = \begin{bmatrix} -7 & 1 \\ 1 & -7 \end{bmatrix}$

i.e., $M_2 = \begin{bmatrix} C_{11} \\ C_{12} \end{bmatrix} = \begin{bmatrix} -7 \\ -1 \end{bmatrix}$

M_1 and M_2 are the eigen vectors corresponding to the eigen values of λ_1 and λ_2 .

5.11. When lag/lead/lag-lead compensation is employed?

Ans:

1. Lag compensation is employed for a stable system for improvement in steady state performance.
2. Lead compensation is employed for stable/unstable system for improvement in transient-state performance.
3. Lag-lead compensation is employed for stable/unstable system for improvement in both steady-state and transient state performance.



B.Tech.
(SEM. V) ODD SEMESTER THEORY
EXAMINATION, 2019-20
CONTROL SYSTEM

Time : 3 Hours

Max Marks : 70

Note: Attempt all sections. If any missing data is required, then choose suitably.

SECTION-A

1. Attempt all parts of the following : (2 × 7 = 14)
 a. The impulse response of unity feedback control system is $C(t) = te^{-t} + 2be^{-t}$, find transfer function.

Ans. Refer Q. 2.12, Page SQ-7C, Unit-2, 2 Marks Questions.

- b. Give example of type zero, type one and type two systems.

Ans. Refer Q. 2.8, Page SQ-6C, Unit-2, 2 Marks Questions.

- c. What is damping constant give its relation with time constant ?

Ans. Refer Q. 2.10, Page SQ-7C, Unit-2, 2 Marks Questions.

- d. What is centroid of asymptotes and how the centroid is calculated ?

Ans. Refer Q. 3.5, Page SQ-9C, Unit-3, 2 Marks Questions.

- e. Sketch polar plot of $G(s) = 1/(1 + \alpha S)$

Ans. Refer Q. 4.7, Page SQ-11C, Unit-4, 2 Marks Questions.

- f. Explain gain crossover frequency margin, phase crossover frequency.

Ans. Refer Q. 4.5, Page SQ-11C, Unit-4, 2 Marks Questions.

- g. Enlist the properties of state transition matrix.

Ans. Refer Q. 5.8, Page SQ-16C, Unit-5, 2 Marks Questions.

SECTION-B

2. Attempt any three parts of the following : (7 × 3 = 21)

- a. State and explain Masson's gain formula. For the system shown in the Fig. 1 find the overall transfer function of system using block diagram reduction.

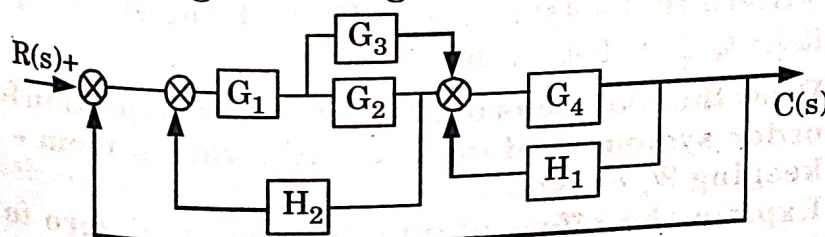


Fig. 1.

Ans. Refer Q. 1.13, Page 1-18C, Unit-1.

b. The close loop transfer function is given by

$$T(s) = k(s+z)/s^2 + 4s + 8, \text{ Where } k, z \text{ is adjustable.}$$

- If $r(t) = t$ finds k and z so that steady error is zero.
- for the value of k, z obtain in part (i) find $e(\infty)$ for input $r(t) = t^2/2$

Ans. Refer Q. 2.17, Page 2-25C, Unit-2.

c. Explain stability on basis of location of poles and zeros. For a unity feedback system $G(s) = k/s(s+1)(1+2s)(1+3s)$. Determine range of k for stability, value of k for frequency of sustain oscillation.

Ans. Refer Q. 3.5, Page 3-7C, Unit-3.

d. Sketch Nyquist plot for $G(s)H(s) = 6/s^2(s+2)$ comment on stability.

Ans. Refer Q. 4.9, Page 4-11C, Unit-4.

e. Explain the term : State, State Space, State Vector. A SISO system has transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^3 + 7s^2 + 14s + 8}$$

write down the state equation and stage diagram.

Ans. Refer Q. 5.16, Page 5-24C, Unit-5.

SECTION-C

3. Attempt any one part of the following : (7 × 1 = 7)

a. Explain the effect of sensitivity on feedback system, Determine sensitivity of negative close loop system where $G(s) = 20/s(s+4)$, $H(s) = 0.5$ wrt forward path transfer function, feedback path transfer function.

Ans. Refer Q. 1.16, Page 1-21C, Unit-1.

b. Derive block diagram of armature controlled and field controlled DC motor with proper labeling of circuit diagram.

Ans. Refer Q. 1.20, Page 1-26C, Unit-1.

4. Attempt any one part of the following : (7 × 1 = 7)

a. Explain steady state error due to step input for type 0, 1 and 2 systems. Find steady state error for $-G(s) = 10(1+4s)/s^2(1+s)$, $H(s) = 1$, input $r(t) = 1 + t + t^2/2$.

Ans. Refer Q. 2.18, Page 2-26C, Unit-2.

b. i. Draw the root locus of characteristics equation for second order system as damping ratio varies from $-\infty$ to $+\infty$ keeping W_n constant.

ii. Explain the effect of adding poles and zero to transfer function.

Ans. i. Refer Q. 2.10, Page 2-15C, Unit-2.

ii. Refer Q. 2.11, Page 2-16C, Unit-2.

5. Attempt any one part of the following :

a. A unity feedback system has an open loop transfer function. (7 × 1 = 7)

Draw the root locus for the system.

$$G(s)H(s) = k(s+2)/(s+3)(s^2+2s+2)$$

Ans. Refer Q. 3.14, Page 3-18C, Unit-3.

b. What is the necessary condition for stability ? Explain limitation of Routh's stability method, construct Routh array and determine the stability of the system whose characteristics equation, $s^5 + 2s^4 + s^3 + 2s^2 + s + 4 = 0$

Ans. Refer Q. 3.9, Page 3-10C, Unit-3.

6. Attempt any one part of the following :

a. Draw Bode Plot of unity feedback control system having (7 × 1 = 7)

$$\text{OLTF } G(s) = \frac{10}{s(1+0.2s)(1+0.2s)}$$

Determine GM, PM, gain cross over frequency, phase cross over frequency and discuss stability of closed loop system.

Ans. Refer Q. 4.17, Page 4-22C, Unit-4.

b. Explain the strengths of frequency response approach, establish correlation between frequency domain response and time domain response.

Ans. Refer Q. 4.20, Page 4-26C, Unit-4.

7. Attempt any one part of the following :

a. What is lag compensator ? What are the characteristics of lag compensation ? Explain the frequency response of lag compensator. (7 × 1 = 7)

Ans. Refer Q. 5.5, Page 5-10C, Unit-5.

b. The state equations are given below; check controllability and observability of a system.

$$\dot{x}_1 = x_2 - x_3 + 3r$$

$$\dot{x}_2 = x_1 + x_2 + x_3 - 2r$$

$$\dot{x}_3 = x_1 + x_2 + r$$

$$y = x_1$$

Ans. Refer Q. 5.24, Page 5-33C, Unit-5.



B.Tech.**(SEM. V) ODD SEMESTER THEORY
EXAMINATION, 2020-21
CONTROL SYSTEM**

Time : 3 Hours

Max. Marks : 100

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION - A

1. Attempt all questions in brief. (2 × 10 = 20)
 - a. List the advantages of closed loop over open loop system.

Ans: Refer Q. 1.2, Page SQ-1C, Unit-1, 2 Marks Questions.
 - b. Define transfer function. Give an example for it.

Ans: Refer Q. 1.10, Page SQ-3C, Unit-1, 2 Marks Questions.
 - c. Write the Mason's gain formula.

Ans: Refer Q. 1.6, Page SQ-2C, Unit-1, 2 Marks Questions.
 - d. What are the standard test signals used in control system?

Ans: Refer Q. 2.1, Page SQ-5C, Unit-2, 2 Marks Questions.
 - e. Distinguish between type and order of a system.

Ans: Refer Q. 2.7, Page SQ-6C, Unit-2, 2 Marks Questions.
 - f. Define a stable system.

Ans: Refer Q. 3.1, Page SQ-8C, Unit-3, 2 Marks Questions.
 - g. Define gain and phase margins.

Ans: Refer Q. 4.5, Page SQ-10C, Unit-4, 2 Marks Questions.
 - h. Why negative feedback is preferred in closed loop control system?

Ans: Refer Q. 1.3, Page SQ-2C, Unit-1, 2 Marks Questions.
 - i. Define state and state variable.

Ans: Refer Q. 5.4, Page SQ-13C, Unit-5, 2 Marks Questions.
 - j. State the necessary condition of stability.

Ans: Refer Q. 3.2, Page SQ-8C, Unit-3, 2 Marks Questions.

SECTION - B

2. Attempt any three of the following : (10 × 3 = 30)
 - a. Determine the transfer function of the given system using block diagram reduction.

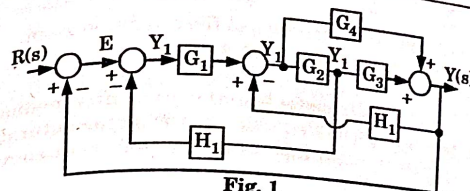


Fig. 1.

Ans: Refer Q. 1.11, Page 1-14C, Unit-1.

- b. Write down the differential equation governing the mechanical translation system and find the transfer function.

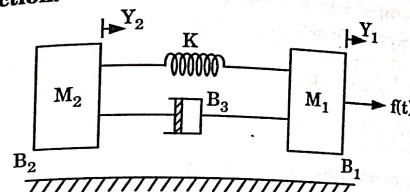


Fig. 2.

Ans: Refer Q. 1.6, Page 1-7C, Unit-1.

- c. Define the following term :
 - i. Rise time
 - ii. Peak time
 - iii. Peak overshoot
 - iv. Settling time

Ans: Refer Q. 2.7, Page 2-11C, Unit-2.

- d. Find the state transition matrix of the following matrix

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

Ans: Refer Q. 5.19, Page 5-28C, Unit-5.

- e. What are the characteristics of servomotors? Compare the AC and DC servomotors.

Ans: Refer Q. 1.21, Page 1-27C, Unit-1.

SECTION - C

3. Attempt any one part of the following : (10 × 1 = 10)
 - a. Derive the expression for steady state error and explain K_p , K_v and K_a .

Ans: Refer Q. 2.13, Page 2-20C, Unit-2.
 - b. For the unity feedback control system the open loop transfer function, $G(s) = 10(s+2)/s^2(s+4)$. Find the e_{ss} when the input is $r(t) = 3 - 2t + 3t^2$. And find K_p , K_v and K_a .

Ans: Refer Q. 2.14, Page 2-23C, Unit-2.
4. Attempt any one part of the following : (10 × 1 = 10)

- a. Derive the time response of first order system.

Ans. Refer Q. 2.2, Page 2-4C, Unit-2.

- b. The open loop transfer function of a unity feedback control system is given by $G(s) = 9/s(s+3)$. Find the natural frequency of response, damping ratio, damped frequency and time constant.

Ans. Refer Q. 2.6, Page 2-10C, Unit-2.

5. Attempt any one part of the following :

- a. Determine the Routh's stability of given characteristics equation $s^4 + 18s^3 + 18s^2 + 16s + 5 = 0$. (10 × 1 = 10)

Ans. Refer Q. 3.7, Page 3-9C, Unit-3.

- b. Draw Nyquist plot for the open loop transfer function is $G(s)H(s) = 1/s(s+2)$.

Ans. Refer Q. 4.8, Page 4-10C, Unit-4.

6. Attempt any one part of the following :

- a. Sketch the root locus of the system, whose transfer function is $G(s) = K(s+15)/s(s+1)(s+5)$. (10 × 1 = 10)

Ans. Refer Q. 3.13, Page 3-16C, Unit-3.

- b. Write down the procedure for designing lag-lead compensator.

Ans. Refer Q. 5.8, Page 5-15C, Unit-5.

7. Attempt any one part of the following :

- a. Find the controllability and observability of the system (10 × 1 = 10)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0[u]$$

Ans. Refer Q. 5.25, Page 5-34C, Unit-5.

- b. Obtain the state space representation for the following

differential equation $\ddot{y} + 5\dot{y} + 7y = 114u$, where 'y' is the output and 'u' is the input.

Ans. Refer Q. 5.12, Page 5-21C, Unit-5.



B.Tech.

(SEM. V) ODD SEMESTER THEORY
EXAMINATION, 2021-22
CONTROL SYSTEM

Time : 3 Hours

Max. Marks : 100

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt all questions in brief. (2 × 10 = 20)

- a. Draw the block diagram, which represent a driver driving a car.

Ans. Refer Q. 1.8, Page SQ-3C, Unit-1, 2 Marks Questions.

- b. Define : Self loop and non-touching loop in signal flow graph by suitable example.

Ans. Refer Q. 1.9, Page SQ-3C, Unit-1, 2 Marks Questions.

- c. What do you mean by settling time, write expression for 2nd order system ?

Ans. Refer Q. 2.3, Page SQ-5C, Unit-2, 2 Marks Questions.

- d. The OLTF of a unity feedback system is $G(s) = 1/s(s+1)(s+4)$. find the steady state error (e_{ss}) due to a unit step.

Ans. Refer Q. 2.9, Page SQ-6C, Unit-2, 2 Marks Questions.

- e. What are the limitations of Routh Hurwitz criterion ?

Ans. Refer Q. 3.4, Page SQ-8C, Unit-3, 2 Marks Questions.

- f. State absolute stability and relative stability.

Ans. Refer Q. 3.3, Page SQ-8C, Unit-3, 2 Marks Questions.

- g. Draw the polar plot $G(s) = 1/(s+2)$

Ans. Refer Q. 4.8, Page SQ-12C, Unit-4, 2 Marks Questions.

- h. Write advantages of Bode Plot.

Ans. Refer Q. 4.10, Page SQ-12C, Unit-4, 2 Marks Questions.

- i. Find the eigen vectors of the matrix

$$A = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$$

Ans. Refer Q. 5.10, Page SQ-14C, Unit-5, 2 Marks Questions.

- j. Why compensators are used in control system, what is effect of lag compensator ?

Refer Q. 5.2, Page SQ-13C, Unit-5, 2 Marks Questions.

SECTION - B

2. Attempt any three of the following : (10 × 3 = 30)
 a. Write down various rules involve in block diagram reduction method.

Refer Q. 1.8, Page 1-9C, Unit-1.

- b. The unity feedback system is characterized by an open loop transfer function is $G(S) = K/s(s + 20)$. Determine the gain K , so that the system will have a damping ratio of 0.6. For this value of K , determine unit step response, time domain specifications: settling time (2% criterion), peak overshoot, rise time, peak time, delay time for a unit-step input.

Refer Q. 2.9, Page 2-13C, Unit-2.

- c. Explain the effect of addition of pole and zero on root locus and time domain specifications.

Refer Q. 3.18, Page 3-24C, Unit-3.

- d. Sketch the polar plot of the following function, also determine gain margin, phase margin, $H(s) = 1$

$$G(s) = \frac{1}{s(1+s)(1+2s)}$$

Refer Q. 4.3, Page 4-5C, Unit-4.

- e. State properties of State Transition Matrix (STM), find out state transition matrix for

$$A = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}$$

Refer Q. 5.20, Page 5-29C, Unit-5.

SECTION - C

3. Attempt any one part of the following : (10 × 1 = 10)
 a. Find out C/R by using block diagram reduction method.

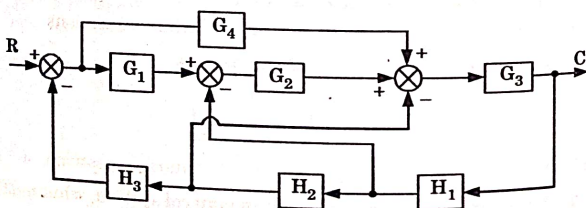


Fig. 1.

Refer Q. 1.12, Page 1-16C, Unit-1.

- b. Find the overall gain of the system whose signal flow graph is shown below.

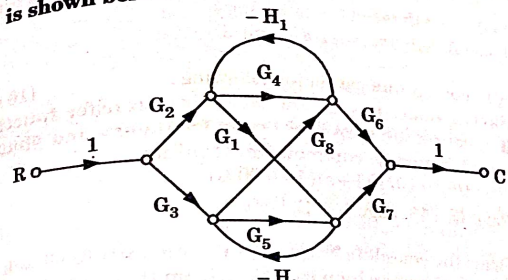


Fig. 2.

Refer Q. 1.15, Page 1-20C, Unit-1.

4. Attempt any one part of the following : (10 × 1 = 10)
 a. Compare proportional (P) control action with integral (I) control action and prove that by using proportional integral PI controller steady state error become zero in a system.

Refer Q. 2.23, Page 2-32C, Unit-2.

- b. Find out various error coefficients : K_p , K_v , K_a and steady state error for standard step, ramp, and parabolic inputs for system shown below :

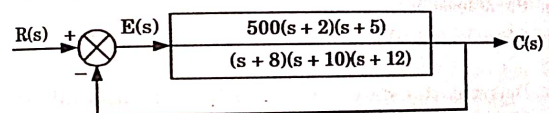


Fig. 3.

Refer Q. 2.15, Page 2-23C, Unit-2.

5. Attempt any one part of the following : (10 × 1 = 10)
 a. Explain the effect of pole location on stability of a system by suitable diagram, determine range of K and frequency of sustained oscillations for a given unity feedback system.

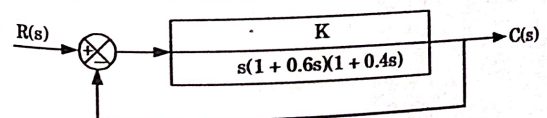


Fig. 4.

Refer Q. 3.8, Page 3-9C, Unit-3.

- b. Sketch the root locus of the system whose open loop transfer function is $G(S) = K/s(s+2)(s+5)$. Find the value of K so that system is marginal stable find out damped frequency of oscillation, also find K when the damping ratio of the closed loop system is 0.5.

Ans. Refer Q. 3.15, Page 3-20C, Unit-3.

6. Attempt any one part of the following :

- a. Sketch Bode plot for the following transfer function and determine the gain cross over frequency and phase cross over frequency, comment on stability.
 $G(s)H(s) = 10/s(1+s)(1+0.002s)$

Ans. Refer Q. 4.18, Page 4-23C, Unit-4.

- b. Draw the complete Nyquist plot for a unity feedback system having the open loop function; from this plot obtain all the information regarding stability.
 $G(s)H(s) = k/s(s+3)(s+5)$

Ans. Refer Q. 4.11, Page 4-13, Unit-4.

7. Attempt any one part of the following :

- a. Design a lead compensator for a system whose open loop transfer function is

$$G(s)H(s) = 4/s(s+2)$$

It will fulfill following requirement

- Static velocity error constant = 20 sec^{-1}
- PM at least 50°
- GM at least 10 db

Ans. Refer Q. 5.3, Page 5-6C, Unit-5.

- b. Determine the state controllability and observability of the system described

$$\dot{x} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 3 \\ -7 & 5 & 9 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \text{ and}$$

$$y = [5 \ 2 \ 7]x$$

Ans. Refer Q. 5.26, Page 5-36C, Unit-5.



B.Tech.

(SEM. V) ODD SEMESTER THEORY
EXAMINATION, 2022-23
CONTROL SYSTEM

Time : 3 Hours

Max. Marks : 100

Note: 1. Attempt all Sections. If require any missing data, then choose suitably.

Section-A

1. Attempt all questions in brief. (2 × 10 = 20)

- a. Define transfer function of a control system.

Ans. Refer Q. 1.10, Page SQ-3C, Unit-1; 2 Marks Questions.

- b. What is Mason's gain formula? Explain.

Ans. Refer Q. 1.6, Page SQ-2C, Unit-1, 2 Marks Questions.

- c. A unity feedback system with open-loop transfer function $G(s) = 2/[s(s+p)]$ is critically damped. Calculate the value of p ?

Ans.

$$\text{Given : } G(s) = \frac{2}{s(s+p)}, H(s) = 1$$

To Find : Value of p .

1. Transfer function of closed loop system,

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{2/s(s+p)}{1 + 2/s(s+p) \times 1} = \frac{2}{s^2 + sp + 2} \quad \dots(1)$$

2. Comparing eq. (1) with

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n^2 = 2$$

$$\omega_n = \sqrt{2}$$

$$2\xi\omega_n = p$$

3. As system is critically damped, $\xi = 1$

$$2 \times 1 \times \sqrt{2} = p$$

$$p = 2\sqrt{2}$$

d. Explain the effects of PD and PI control action on a second order system performance.

Ans.

A. Effects of PD control :

1. It increases damping ratio.
2. It reduces settling time.
3. Type of the system remains unchanged.

B. Effects of PI control :

1. It increases order of the system.
2. It increases type of the system.

e. Explain the special cases of Routh Hurwitz criterion.

Ans. Refer Q. 3.4, Page 3-6C, Unit-3.

f. Define the centroid and breakaway point.

Ans.

A. Centroid : Refer Q. 3.5, Page SQ-9C, Unit-3, 2 Marks Questions.

B. Breakaway point : Refer Q. 3.6, Page SQ-9C, Unit-3, 2 Marks Questions.

g. Define relative stability of the system.

Ans. Refer Q. 3.3, Page SQ-8C, Unit-3, 2 Marks Questions.

h. Explain gain margin and phase margin in terms of Bode plot.

Ans. Refer Q. 4.13, Page 4-15C, Unit-4.

i. Define state variable and state vector.

Ans. Refer Q. 5.9, Page 5-16C, Unit-5.

j. Mention any four properties of state transition matrix.

Ans. Refer Q. 5.18, Page 5-27C, Unit-5.

Section-B

2. Attempt any three of the following : (10 × 3 = 30)

a. Find $C(s)/R(s)$ using Mason's gain formula for the given signal flow graph given below.

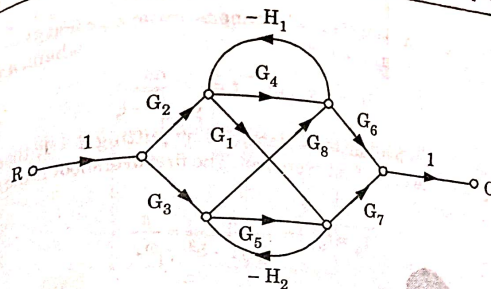


Fig. 1.

Ans. Refer Q. 1.15, Page 1-20C, Unit-1.

b. With neat sketch explain all time domain specifications for a second order control system and derive the formula for peak time.

Ans.

A. Time domain specifications : Refer Q. 2.7, Page 2-11C, Unit-2.

B. Derivation of peak time :

1. We may obtain the peak time by differentiating $C(t)$ with respect to time and letting this derivative equal to zero. Since

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin[\omega_d t + \phi]$$

2. After differentiating $C(t)$, we get

$$\frac{dC(t)}{dt} = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cos[\omega_d t + \phi] \omega_d + \sin[(\omega_d t + \phi)] \frac{\xi\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \dots (1)$$

3. For maximum value of

$$\frac{dC(t)}{dt} = 0$$

4. Eq. (1) can be written as,

$$\cos[\omega_d t + \phi] \omega_d = \sin[\omega_d t + \phi] \xi \omega_n \quad [\because \omega_d = \omega_n \sqrt{1-\xi^2}]$$

$$\text{or } \cos\left[(\omega_n \sqrt{1-\xi^2})t + \phi\right] \sqrt{1-\xi^2} = \sin\left[(\omega_n \sqrt{1-\xi^2})t + \phi\right] \xi \dots (2)$$

5. Put $\xi = \cos \phi$ and $\sqrt{1-\xi^2} = \sin \phi$, eq. (2) now becomes

$$\cos(\omega_d t + \phi) \sin \phi = \sin(\omega_d t + \phi) \cos \phi$$

$$\text{or } \sin(\omega_d t + \phi) \cos \phi - \cos(\omega_d t + \phi) \sin \phi = 0$$

$$\text{i.e., } \sin \omega_d t = 0$$

6. We have the solution $t = \infty$ (time to various peaks) and $\omega_d t = n\pi$ where, $n = 0, 1, 2, \dots$
- $$\therefore (\omega_n \sqrt{1 - \xi^2}) t = n\pi \quad \text{or} \quad t = \frac{n\pi}{\omega_n \sqrt{1 - \xi^2}} \quad \dots(3)$$
7. Maximum overshoot is identified by putting $n = 1$, therefore the peak time to the first overshoot. The first overshoot is the maximum overshoot.

$$t_p = t_{\max} = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}} = \frac{\pi}{\omega_d} \quad \dots(4)$$

- c. What do you mean by the root locus? State and explain all the steps used for drawing the root locus plot.

Ans:

A. Root locus: Refer Q. 3.10, Page 3-11C, Unit-3.

B. Procedure of root locus: Refer Q. 3.11, Page 3-12C, Unit-3.

- d. Sketch the polar plot and determine the gain cross-over frequency, phase cross-over frequency, gain margin and phase margin with given open loop transfer function given below. Also comment on the stability of the system.

$$G(s) = \frac{1}{s(1+s)(1+2s)}$$

Ans: Refer Q. 4.3, Page 4-5C, Unit-4.

- e. Find the state transition matrix for matrix A given below:

$$A = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}$$

Ans: The procedure is same as Q. 5.19, Page 5-28C, Unit-5.
[Ans. State transition matrix,

$$\phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & -e^{-t} + e^{-2t} \\ 2(e^{-t} - e^{-2t}) & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

Section-C

3. Attempt any one part of the following:

- a. Explain open loop and closed loop control system with a suitable example in each case.

Ans: Refer Q. 1.2, Page 1-3C, Unit-1.

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- b. Using block reduction technique calculate transfer function of the block diagram given below:

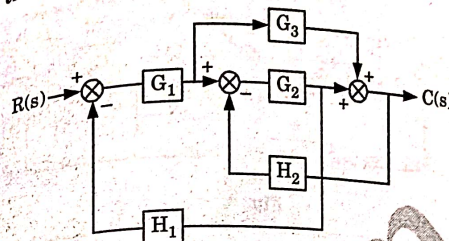


Fig. 2.

Step-1: Shifting take off point before G_2 .

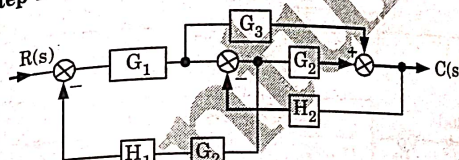


Fig. 3.

Step-2: Shifting take off point before summing point.

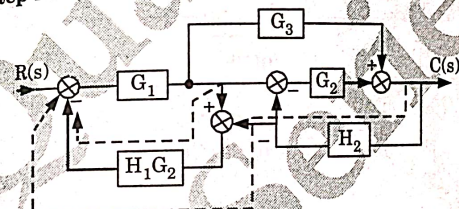


Fig. 4.

Step-3: Separating the paths in the feedback path shown dotted,

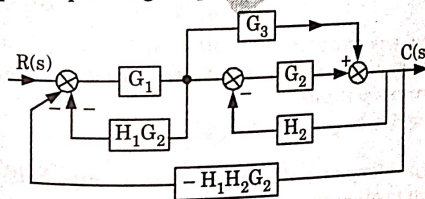


Fig. 5.

Step-4: Shifting summing point and then interchanging the two summing points.

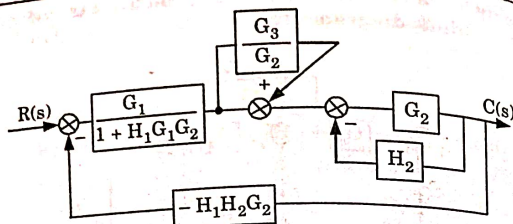


Fig. 6.

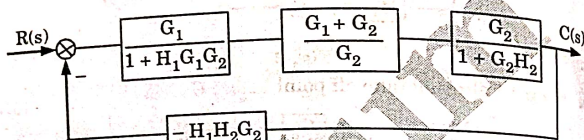


Fig. 7.

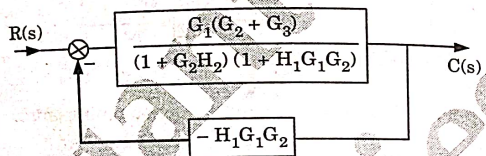


Fig. 8.

$$\frac{C(s)}{R(s)} = \frac{G_1(G_2 + G_3)}{(1 + G_2H_2)(1 + H_1G_1G_2)} \cdot \frac{(-H_1H_2G_2)}{1 + \frac{G_1(G_2 + G_3)(-H_1H_2G_2)}{(1 + G_2H_2)(1 + H_1G_1G_2)}}$$

$$= \frac{G_1(G_2 + G_3)}{1 + G_2H_2 + H_1G_1G_2 - G_1G_2G_3H_1H_2}$$

4. Attempt any one part of the following : (10 × 1 = 10)

a. Derive the time response of a second order control system for unit step input.

Ans. Refer Q. 2.4, Page 2-6C, Unit-2.

b. The open loop transfer function of a unity feedback control system is given by,

$$G(s) = \frac{K}{s(sT + 1)}$$

The system input is a unit step function. Determine :

- By what factor amplifier gain K should be multiplied so that the damping ratio is increased from 0.3 to 0.9.
- By what factor the time constant T should be multiplied so that the damping ratio is reduced from 0.8 to 0.2.

Ans.

Given, $G(s) = \frac{K}{s(sT + 1)}, H(s) = 1$

1. The closed-loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{\frac{K}{s(sT + 1)}}{1 + \frac{K}{s(sT + 1)}} = \frac{K}{s(sT + 1) + K}$$

$$= \frac{K}{s^2T + s + K}$$

$$= \frac{K/T}{s^2 + \frac{s}{T} + \frac{K}{T}}$$

2. Comparing the characteristic equation $s^2 + \frac{s}{T} + \frac{K}{T} = 0$, with the standard form other characteristic equation $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ of a second order system,

$$\omega_n^2 = \frac{K}{T}$$

$$\omega_n = \sqrt{\frac{K}{T}}$$

$$2\zeta\omega_n = \frac{1}{T}$$

$$2\zeta \sqrt{\frac{K}{T}} = \frac{1}{T}$$

$$\zeta = \frac{1}{2\sqrt{KT}}$$

$$\zeta = \zeta_1 = 0.3, \text{ let } K = K_1$$

$$\zeta = \zeta_2 = 0.9, \text{ let } K = K_2$$

$$\frac{\zeta_1}{\zeta_2} = \frac{0.3}{0.9} = \frac{1}{3} = \frac{\frac{1}{2\sqrt{K_1T}}}{\frac{1}{2\sqrt{K_2T}}}$$

i. When,
When,

$$\sqrt{\frac{K_2}{K_1}} = \frac{1}{3}$$

$$\frac{K_2}{K_1} = \frac{1}{9}$$

$$K_2 = \frac{1}{9} K_1$$

Hence the gain K_1 at which $\xi = 0.3$ should be multiplied by $\frac{1}{9}$ to increase the damping ratio from 0.3 to 0.9

ii. When, $\xi = \xi_1 = 0.8$, let $T = T_1$

When, $\xi = \xi_2 = 0.2$, let $T = T_2$

$$\frac{\xi_1}{\xi_2} = \frac{0.8}{0.2} = \frac{1}{2\sqrt{K_1 T}} \times 2\sqrt{K T_2}$$

$$\sqrt{\frac{T_2}{T_1}} = 4$$

$$\frac{T_2}{T_1} = 16$$

$$T_2 = 16 T_1$$

Hence the original time constant T_1 should be multiplied by 16 to reduce the damping ratio from 0.8 to 0.2

5. Attempt any one part of the following : (10 × 1 = 10)
a. Sketch the root locus plot and give the stability conditions for the system when open loop transfer function is given by

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)(s+3)}$$

Ans:

Given : $G(s)H(s) = \frac{K}{s(s+1)(s+2)(s+3)}$

To Sketch : Root locus

To determine : Stability

- The open loop poles are at $s = 0, s = -1, s = -2, s = -3$
- There is no open loop zeros.
- Number of poles, $P = 4$
Number of zeros, $Z = 0$
 $P - Z = 4 - 0 = 4$, i.e., four branches of root locus end at infinity.

4. Angle of asymptotes :

$$\theta_q = \frac{(2q+1) \times 180^\circ}{P-Z}$$

[where, $q = 0, 1, 2, 3, \dots, (P-Z-1)$]

$$\theta_0 = \frac{(2 \times 0 + 1) \times 180^\circ}{(4-0)} = 45^\circ$$

$$\theta_1 = \frac{(2 \times 1 + 1) \times 180^\circ}{4-0} = 135^\circ$$

$$\theta_2 = \frac{(2 \times 2 + 1) \times 180^\circ}{(4-0)} = 225^\circ$$

$$\theta_3 = \frac{(2 \times 3 + 1) \times 180^\circ}{(4-0)} = 315^\circ$$

5. Centroid of asymptotes :

$$\sigma = \frac{\sum \text{Real parts of poles} - \sum \text{Real parts of zeros}}{P-Z}$$

$$= \frac{(0-1-2-3)-(0)}{4-0}$$

$$= \frac{-6}{4} = -1.5$$

6. Breakaway points :

The characteristic equation is,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+1)(s+2)(s+3)} = 0$$

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0$$

$$K = -s^4 - 6s^3 - 11s^2 - 6s$$

$$\frac{dK}{ds} = -4s^3 - 18s^2 - 22s - 6 = 0$$

...(1)

$$4s^3 + 18s^2 + 22s + 6 = 0$$

After solving eq. (1), we get

$$s = -0.381, -0.2619$$

Both are valid breakaway points.

7. Intersection points with imaginary axis :

The characteristic equation, $1 + G(s)H(s) = 0$

$$s^4 + 6s^3 + 11s^2 + 6s + K = 0$$

Routh array,

s^4	1	11	K
s^3	6	6	0
s^2	10	K	0
s^1	$\frac{60-6K}{10}$	0	
s^0	K		

The value of K at imaginary axis,

$$\frac{60-6K}{10} = 0$$

$$K_{\text{mar}} = 10$$

Auxiliary equation,

$$10s^2 + K = 0$$

$$10s^2 + 10 = 0$$

$$s^2 = -1$$

$$s = \pm j$$

8. For $0 < K < 10$, system is absolutely stable. At $K = 0$, system is marginally stable and for $K > 10$ system is unstable.

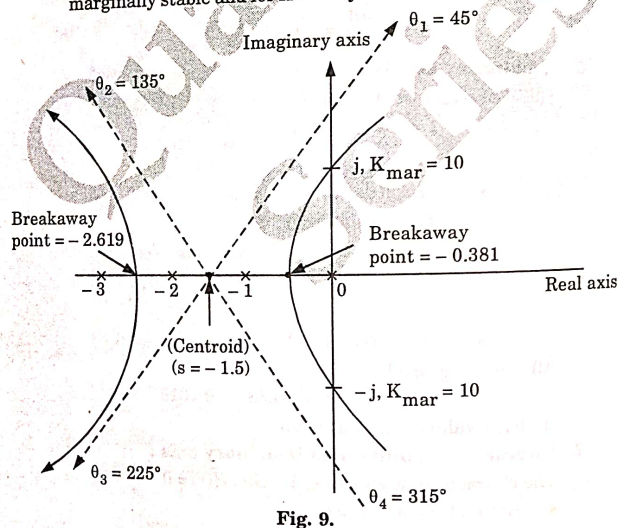


Fig. 9.

- b. The open loop T.F. of certain unity feedback system is

$$G(s) = \frac{K(s+1)}{s(s-1)(s+6)}$$

Determine

- Range of K for stability
- Marginal value of K
- Location of roots for marginal stability.

Ans.

$$\text{Given: } G(s) = \frac{K(s+1)}{s(s-1)(s+6)}, H(s) = 1$$

- To Find:
- Range of K
 - Marginal value of K
 - Location of roots

1. Characteristic equation, $1 + G(s)H(s) = 0$

$$1 + \frac{K(s+1)}{s(s-1)(s+6)} \times 1 = 0$$

$$s(s-1)(s+6) + K(s+1) = 0$$

$$s^3 + 5s^2 - (6-K)s + K = 0$$

The Routh table is,

s^3	1	$-(6-K)$
s^2	5	K
s^1	$\frac{-5(6-K)-K}{5}$	0
s^0	K	

- i. For stability $K > 0$ and $\frac{-5(6-K)-K}{5} > 0$

$$\text{so } K > 7.5$$

- ii. For marginally stable,

$$\frac{-5(6-K)-K}{5} = 0$$

$$K = 7.5$$

- iii. Auxiliary equation,

$$A(s) = 5s^2 + K = 0$$

[Here, $K = 7.5$]

$$5s^2 + 7.5 = 0$$

$$s = \pm j 1.22$$

The location in s -plane is $(0 \pm j 1.22)$

6. Attempt any one part of the following : (10 × 1 = 10)
 a. Define resonant peak (M_r) resonant frequency (ω_r) and derive the formula for them.

Ans.
 A. Resonant peak and resonant frequency : Refer Q. 4.19, Page 4-25C, Unit-4.
 B. Derivation : Refer Q. 4.20, Page 4-26C, Unit-4.

- b. Draw the Bode plot for the open loop T.F.

$$G(s) = \frac{1000}{s(1 + 0.1s)(1 + 0.001s)}$$

and from the graph determine :

- i. Gain cross-over frequency ii. Phase cross-over frequency
 iii. Gain margin iv. Phase margin
 v. Stability of the system.

Ans.

$$G(s) = \frac{1000}{s(1 + 0.1s)(1 + 0.001s)}$$

To draw : Bode plot

- To Find : i. Gain cross-over frequency
 ii. Phase cross-over frequency
 iii. Gain margin
 iv. Phase margin
 v. Stability of the system.

1. Put $s = j\omega$,

$$G(j\omega) = \frac{1000}{j\omega(1 + 0.1j\omega)(1 + 0.001j\omega)}$$

2. First corner frequency, $\omega_{c1} = 10$ rad/sec
 3. Second corner frequency, $\omega_{c2} = 10^3$ rad/sec
 4. Magnitude plot :

Table 1

Term	Corner frequency	Slope	Change in slope
$\frac{1000}{j\omega}$	—	-20 dB/dec	—
$\frac{1}{1 + 0.1j\omega}$	10 rad/sec	-20 dB/dec	-40 dB/dec
$\frac{1}{1 + 0.001j\omega}$	1000 rad/sec	-20 dB/dec	-60 dB/dec

5. Assume

$$\omega_l = 1 \text{ rad/sec}, \omega_h = 10^4 \text{ rad/sec}$$

$$\text{Gain (dB)}, G_{\omega l} = 20 \log_{10} \left(\frac{1000}{j\omega} \right)_{\omega=\omega_l} = 20 \log_{10} \left(\frac{1000}{1} \right) = 60 \text{ dB}$$

$$G_{\omega c1} = 20 \log_{10} \left(\frac{1000}{j\omega} \right)_{\omega=\omega_{c1}} = 20 \log_{10} \left(\frac{1000}{10} \right) = 40 \text{ dB}$$

$$G_{\omega c2} = \left[\text{Slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right] + G_{\omega c1}$$

$$= -40 \times \log_{10} \left(\frac{\omega_{c2}}{\omega_{c1}} \right) + G_{\omega c1}$$

$$= -40 \log_{10} \left(\frac{1000}{10} \right) + (40) = -40 \text{ dB}$$

$$G_{\omega h} = \left[\text{Slope from } \omega_h \text{ to } \omega_{c2} \times \log \frac{\omega_h}{\omega_{c2}} \right] + G_{\omega c2}$$

$$= -60 \times \log \left(\frac{\omega_h}{\omega_{c2}} \right) + G_{\omega c2}$$

$$= -60 \times \log \left(\frac{10^4}{10^3} \right) + (-40) = -100 \text{ dB}$$

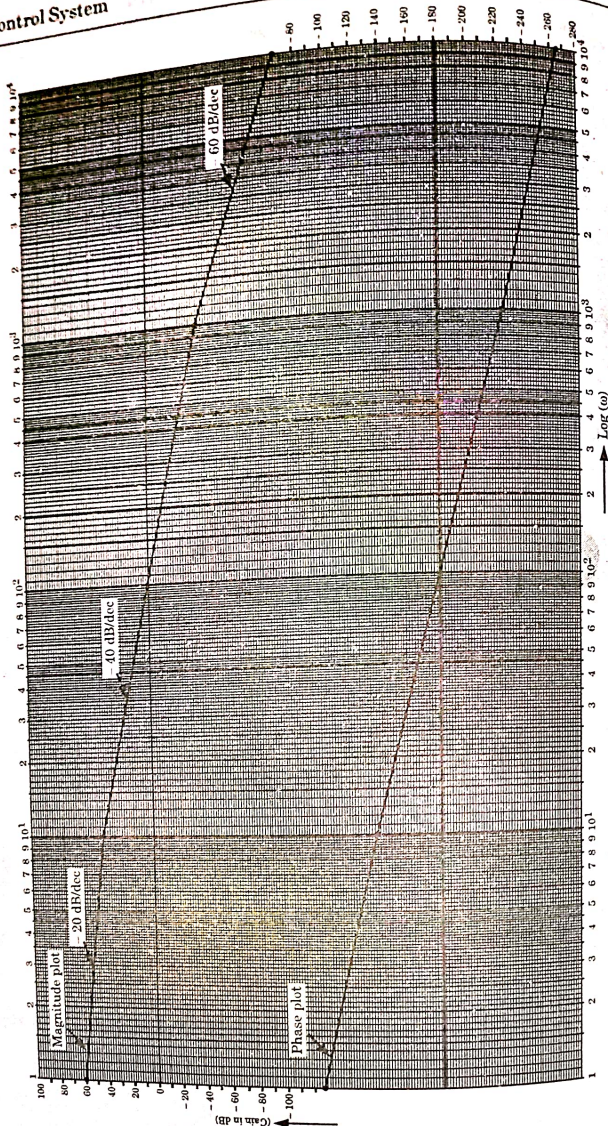
6. Phase plot : $\phi = \angle G(j\omega)H(j\omega)$
 $= -90 - \tan^{-1}(0.1\omega) - \tan^{-1}(0.001\omega)$

Table 2

ω	1	10	10^3	10^4
ϕ	-95.7°	-135.6°	-224.4	-264.2

8. Results from Bode plot :

- i. Gain cross-over frequency, $\omega_{gc} = 10^2$ rad/sec
 ii. Phase cross-over frequency, $\omega_{pc} = 10^2$ rad/sec
 iii. Gain margin = 0 dB
 iv. Phase margin = 0°
 v. **Stability** : The gain margin and phase margin both are zero therefore, the system is marginally stable.



7. Attempt any one part of the following : (10 × 1 = 10)
- a. What are the different types of compensators used in control system ? Discuss lead compensator and derive the relation between maximum lead angle ϕ_m and α .

Ans. A. Different types of compensators : There are three types of compensators used in control system :

1. **Lag compensator** : The lag compensator is an electrical network which produces a sinusoidal output having the phase lag when a sinusoidal input is applied.
2. **Lead compensator** : The lead compensator is an electrical network which produces a sinusoidal output having phase lead when a sinusoidal input is applied.
3. **Lag-lead compensator** : Lag-lead compensator is an electrical network which produces phase lag at one frequency region and phase lead at other frequency region. It is a combination of both the lag and the lead compensators.

b. **Lead compensator and the relation between ϕ_m and α** : Refer Q. 5.1, Page 5-2C, Unit-5.

- b. Examine the controllability and observability of a system with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, C = [10 \ 5 \ 1]$$

Ans. Controllability test :

1. We have, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -12 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -12 \end{bmatrix} = \begin{bmatrix} 1 \\ -12 \\ 61 \end{bmatrix}$$

2. Controllability test matrix is given by, $U = [B : AB : A^2B]$

$$U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -12 \\ 1 & -12 & 61 \end{bmatrix}$$

$$|U| = 1 \begin{vmatrix} 1 & -12 \\ -12 & 61 \end{vmatrix} - 0 \begin{vmatrix} 0 & -12 \\ 1 & 61 \end{vmatrix} + 1 \begin{vmatrix} 0 & 1 \\ 1 & -12 \end{vmatrix}$$

$$\therefore |U| \neq 0$$

3. Therefore, the rank of U is equal to its order i.e., 3 ($n=3$) indicating that the system is controllable.

Observability test :

1. We have, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$, $A^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}$, $C^T = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}$

$$A^T C^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -1 \\ -1 \end{bmatrix}$$

$$(A^T)^2 C^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} -6 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 5 \end{bmatrix}$$

2. Observability test matrix is given by,

$$V = [C^T : A^T C^T : (A^T)^2 C^T]$$

$$\therefore V = \begin{bmatrix} 10 & -6 & 6 \\ 5 & -1 & 5 \\ 1 & -1 & 5 \end{bmatrix}$$

$$V = 10 \begin{vmatrix} -1 & 5 \\ -1 & 5 \end{vmatrix} - (-6) \begin{vmatrix} 5 & 5 \\ 1 & 5 \end{vmatrix} + 6 \begin{vmatrix} 5 & -1 \\ 1 & -1 \end{vmatrix}$$

$$= 10[(-1)5 - 5(-1)] + 6(5 \times 5 - 5 \times 1) + 6[(5)(-1) - (-1)(1)]$$

$$= 10 \times 0 + 6 \times 20 + 6 \times (-4) = 96$$

3. Observability matrix $V \neq 0$, hence its rank i.e., 3 equal order i.e., 3, therefore, system is completely observable.



B. Tech.
(SEM. V) ODD SEMESTER THEORY
EXAMINATION, 2023-24
CONTROL SYSTEM

Time : 3 Hours

Max. Marks : 100

Note: 1. Attempt all sections. If require any missing data; then choose suitably.

Section-A

1. Attempt all questions in brief.
a. Explain the properties of signal flow graph. (2 × 10 = 20)

Ans:

1. **Directionality** : Signal flow graphs exhibit directed edges representing the flow of signals or information between nodes, indicating the causal relationships within the system.
2. **Modularity** : Signal flow graphs allow for modular representation of complex systems, simplifying analysis by breaking down the system into interconnected subsystems or components.

- b. Define transfer function of a control system.

Ans: Refer Q. 1.10, Page SQ-3C, Unit-1, 2 Marks Questions.

- c. Mention the effect of ξ on second order system performance for unit step input when

- i. $\xi = 0$, ii. $0 < \xi < 1$,
iii. $\xi = 1$, iv. $1 < \xi < \infty$

Ans: In a second-order system with a unit step input, the damping ratio ξ affects performance as follows :

- i. When $\xi = 0$: Undamped, oscillatory response.
ii. When $0 < \xi < 1$: Underdamped, oscillations with exponential decay.
iii. When $\xi = 1$: Critically damped, fastest response without oscillations.
iv. When $1 < \xi < \infty$: Overdamped, slow response without oscillations.

- d. What are the standard test signals ? Explain.

Ans: Refer Q. 2.1, Page 2-2C, Unit-2.

- e. On the basis of bounded input bounded output stability define stable system and unstable system.

Ans. 1. **Stable system** : A stable system is one in which, for any bounded input signal, the output of the system remains bounded as well.

2. **Unstable system** : An unstable system is one where there exists at least one bounded input signal that results in an unbounded output signal.

- f. List the disadvantages of Routh-Hurwitz criterion.

Ans. 1. Only indicates stability, not degree of stability.
2. Cannot determine the exact location of system poles.

- g. Describe the resonant peak and resonant frequency.

Ans. Refer Q. 4.19, Page 4-25C, Unit-4.

- h. Explain the minimum and non-minimum phase system.

Ans. A. **Minimum phase system** : A minimum phase system has all its poles and zeros in the left half of the s-plane, ensuring stable and predictable behavior.

B. **Non-minimum phase system** : A non-minimum phase system has at least one zero in the right half of the s-plane, leading to an initial response opposite to the steady-state, potentially causing instability and undesired performance.

- i. List the advantages of state variable approach for the analysis of a control system.

Ans. 1. Facilitates analysis of multivariable systems.
2. Enables straightforward controller design.
3. Provides a unified framework for linear and nonlinear systems.

- j. Define state space and state trajectory.

Ans. A. **State space** : Refer Q. 5.5, Page SQ-14C, Unit-5, 2 Marks Questions.

- B. **State trajectory** : The state trajectory refers to the path followed by the system's state variables over time in response to inputs or initial conditions. It describes how the system's internal state evolves from an initial state to a final state under the influence of inputs.

Section-B

2. Attempt any three of the following :
a. Determine the transfer function for the block diagram given below by Mason's gain formula. (10 × 3 = 30)

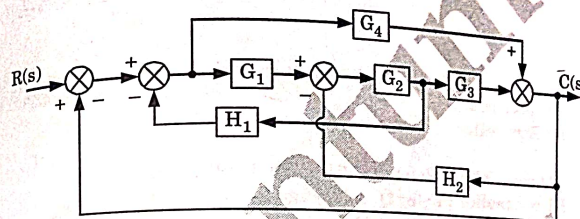


Fig. 1.

Ans.

1. The corresponding equivalent signal flow graph is drawn in Fig. 2.

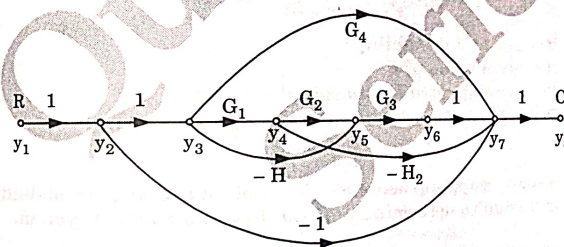


Fig. 2.

2. The gain of the forward paths,

$$P_1 = G_1 G_2 G_3$$

$$P_2 = G_4$$

3. The gain of the individual loops,

$$L_1 = -G_1 G_2 H_1, \quad L_2 = -G_1 G_2 G_3$$

$$L_3 = -G_2 G_3 H_2, \quad L_4 = -G_4$$

$$L_6 = G_4 H_2 G_2 H_1 = G_2 G_4 H_1 H_2$$

4. Calculation of Δ_1 , Δ_2 and Δ ,

$$\Delta_1 = 1 - 0 = 1$$

$$\Delta_2 = 1 - 0 = 1$$

$$\Delta = 1 + G_1 G_2 H_1 + G_1 G_2 G_3 + G_2 G_3 H_2 + G_4 - G_2 H_4 H_1 H_2$$

5. **Transfer function** : Applying Mason's gain formula so as to find C/R , we get

$$T = \frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 + G_4}{1 + G_1 G_2 H_1 + G_1 G_2 G_3 + G_2 G_3 H_2 + G_4 - G_2 H_4 H_1 H_2}$$

- b. Explain the following terms in detail.

- a. PD controller
b. PI controller

Ans: a. PD controller : Refer Q. 2.21, Page 2-29C, Unit-2.

b. PI controller : Refer Q. 2.22, Page 2-30C, Unit-2.

- c. The open loop TF of certain unity feedback system is

$$G(s) = \frac{K(s+1)}{s(s-1)(s+6)} \text{ Determine.}$$

- i. Range of K for stability
ii. Marginal value of K
iii. Location of roots for marginal stability

Ans: Refer Q. 5(b), Page SP-24C, Solved Paper (2022-23).

- d. What is mapping theorem? Also explain the Nyquist stability criterion to determine the stability of a control system.

Ans:

- A. Mapping theorem :

1. The mapping theorem is a fundamental concept in control theory.
2. It describes the relationship between the behavior of a system in the time domain and its behavior in the frequency domain.
3. The mapping theorem is particularly useful in analyzing the stability of linear time-invariant (LTI) systems.

- B. Nyquist stability criterion : Refer Q. 4.6, Page 4-8C, Unit-4.

- e. Determine the transfer function from the state model given below :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Ans:

$$\text{Given : } A = \begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = [1 \quad 1]$$

To Find : Transfer function, $\frac{Y(s)}{U(s)}$

$$1. \quad \frac{Y(s)}{U(s)} = C[sI - A]^{-1}B$$

$$2. \quad [sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} s+3 & -1 \\ 0 & s+1 \end{bmatrix}$$

$$3. \quad [sI - A]^{-1} = \frac{1}{\begin{vmatrix} s+3 & -1 \\ 0 & s+1 \end{vmatrix}} \begin{bmatrix} s+1 & 1 \\ 0 & s+3 \end{bmatrix} = \frac{1}{(s+1)(s+3)} \begin{bmatrix} s+1 & 1 \\ 0 & s+3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+1}{s^2+4s+3} & \frac{1}{s^2+4s+3} \\ 0 & \frac{s+3}{s^2+4s+3} \end{bmatrix}$$

$$4. \quad C[sI - A]^{-1}B = [1 \quad 1] \begin{bmatrix} \frac{s+1}{s^2+4s+3} & \frac{1}{s^2+4s+3} \\ 0 & \frac{s+3}{s^2+4s+3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= [1 \quad 1] \begin{bmatrix} \frac{s+2}{s^2+4s+3} \\ \frac{s+3}{s^2+4s+3} \end{bmatrix} = \frac{2s+5}{s^2+4s+3}$$

$$\frac{Y(s)}{U(s)} = \frac{2s+5}{s^2+4s+3}$$

Section-C

3. Attempt any one part of the following : (10 × 1 = 10)
- a. What do you understand by open loop and closed loop control systems? Discuss the comparative statements between open loop and closed loop control systems. Also explain the practical examples of open loop and closed loop control systems.

Ans. Refer Q. 1.2, Page 1-3C, Unit-1.

- b. Determine $C(S)/R(S)$ for the following system using block reduction method.

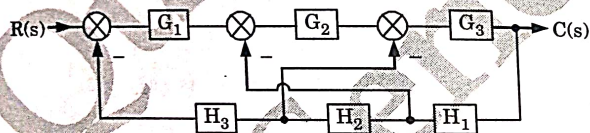


Fig. 3.

Ans.

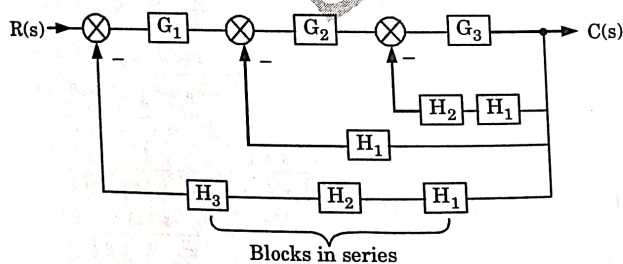


Fig. 4.

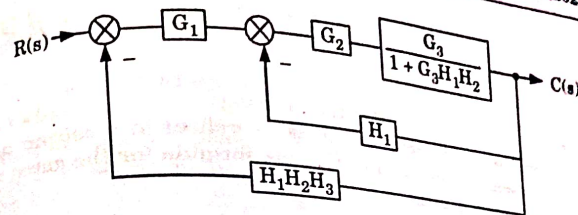


Fig. 5.

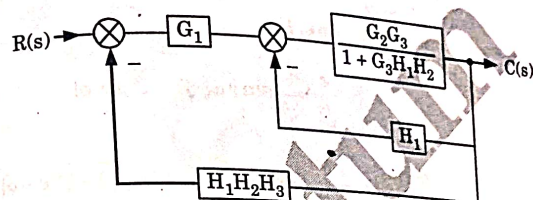


Fig. 6.

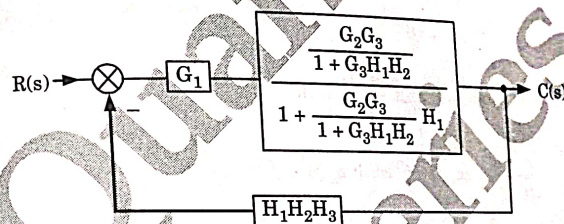


Fig. 7.

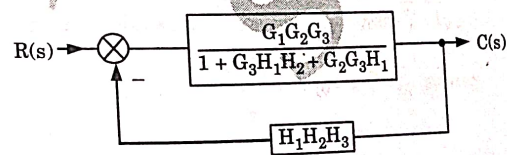


Fig. 8.

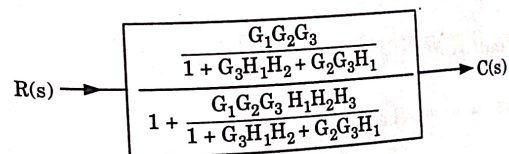


Fig. 9.

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_3 H_1 H_2 + G_2 G_3 H_1 + G_1 G_2 G_3 H_1 H_2 H_3}$$

4. Attempt any one part of the following : (10 × 1 = 10)
 a. Define peak time and peak overshoot of a second order control system and derive the formula for the same with unit step input.

Ans:
 A. Peak time and peak (maximum) overshoot : Refer Q. 2.7, Page 2-11C, Unit-2.

B. Derivation of peak time, t_p :

$$1. \quad c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin[(\omega_n \sqrt{1-\xi^2})t + \phi]$$

$$\therefore \frac{dc(t)}{dt} = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \omega_n \sqrt{1-\xi^2} \cos[(\omega_n \sqrt{1-\xi^2})t + \phi]$$

$$- \frac{(-\xi\omega_n) e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin[(\omega_n \sqrt{1-\xi^2})t + \phi]$$

2. For maximum, Put $\frac{dc(t)}{dt} = 0$

$$\therefore \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \{-\omega_n \sqrt{1-\xi^2} \cos[(\omega_n \sqrt{1-\xi^2})t + \phi] + \xi\omega_n \sin[(\omega_n \sqrt{1-\xi^2})t + \phi]\} = 0$$

$$\therefore \omega_n \sqrt{1-\xi^2} \cos[(\omega_n \sqrt{1-\xi^2})t + \phi] = \xi\omega_n \sin[(\omega_n \sqrt{1-\xi^2})t + \phi]$$

$$\therefore \tan[(\omega_n \sqrt{1-\xi^2})t + \phi] = \frac{\sqrt{1-\xi^2}}{\xi}$$

3. Putting, $\phi = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$

$$\tan\left[(\omega_n \sqrt{1-\xi^2})t + \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}\right] = \frac{\sqrt{1-\xi^2}}{\xi} \quad \dots(1)$$

4. The general solution of eq. (1) is

$$(\omega_n \sqrt{1-\xi^2})t = n\pi$$

where $n = 0, 1, 2, \dots$

...(2)

$$\text{At } n = 1, \text{ peak time } t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

C. Derivation of peak overshoot :

1. Putting $t = t_p$ in $c(t)$,

$$c(t)_{\max} = 1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin[(\omega_n \sqrt{1-\xi^2})t_p + \phi]$$

$$= 1 - \frac{e^{-\xi\omega_n \frac{\pi}{\omega_n \sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left[(\omega_n \sqrt{1-\xi^2}) \frac{\pi}{\omega_n \sqrt{1-\xi^2}} + \phi\right]$$

$$= 1 - \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin(\pi + \phi) = 1 - \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} (-\sin \phi)$$

$$\text{As, } \phi = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi} \therefore \sin \phi = \frac{\sqrt{1-\xi^2}}{\sqrt{1+\xi^2}}$$

$$\text{Hence, } c(t)_{\max} = 1 + \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sqrt{1-\xi^2}$$

$$2. \quad c(t)_{\max} = 1 + e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \sqrt{1-\xi^2}$$

$$M_p = c(t)_{\max} - 1$$

$$M_p = \left(1 + e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\right) - 1$$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\% M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100$$

b. The maximum overshoot for a unity feedback control system having its forward path transfer as $G(s) = \frac{K}{s(sT+1)}$ is to be reduce from 75 % to 25 %. The system input is a unit

step function. Determine the factor by which K should be reduced to achieve the above reduction.

Given,

$$G(s) = \frac{K}{s(sT+1)}$$

$$TF = \frac{\frac{K}{s(sT+1)}}{1 + \frac{K}{s(sT+1)}} \times 1$$

$$\begin{aligned} &= \frac{\frac{K}{s(sT+1)}}{\frac{s(sT+1)+K}{s(sT+1)}} = \frac{K}{s^2T + s + K} \\ &= \frac{K/T}{s^2 + \frac{s}{T} + \frac{K}{T}} \end{aligned} \quad \dots(1)$$

Comparing eq. (1) with

$$TF = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \frac{K}{T} \Rightarrow \omega_n = \sqrt{\frac{K}{T}} \quad \dots(2)$$

$$2\xi\omega_n = \frac{1}{T} \quad \dots(3)$$

Given, $M_{P_1} = 75\%$, $M_{P_2} = 25\%$

M_{P_1} is given by,

$$M_{P_1} = e^{-\frac{\xi_1\pi}{\sqrt{1-\xi_1^2}}} \times 100$$

$$75 = e^{-\frac{\xi_1\pi}{\sqrt{1-\xi_1^2}}} \times 100$$

$$\frac{-\xi_1\pi}{e^{\sqrt{1-\xi_1^2}}} = \frac{75}{100} = 0.75$$

$$\frac{-\xi_1\pi}{\sqrt{1-\xi_1^2}} = \ln(0.75)$$

$$-\frac{\xi_1\pi}{\sqrt{1-\xi_1^2}} = -0.287$$

$$-\xi_1\pi = -0.287\sqrt{1-\xi_1^2}$$

Square on both sides in eq. (4), we get

$$\xi_1^2\pi^2 = 0.0823(1-\xi_1^2) \quad \dots(4)$$

$$\xi_1^2[\pi^2 + 0.0823] = 0.0823$$

$$\xi_1^2 = 8.279 \times 10^{-3} = 0.00827$$

$$\xi_1 = 0.090$$

Also, $M_{P_2} = 25\%$

$$25 = e^{-\frac{\xi_2\pi}{\sqrt{1-\xi_2^2}}} \times 100$$

$$\frac{-\xi_2\pi}{\sqrt{1-\xi_2^2}} = \ln(0.25)$$

$$-\frac{\xi_2\pi}{\sqrt{1-\xi_2^2}} = -1.38$$

$$-\xi_2\pi = (-1.38)\sqrt{1-\xi_2^2}$$

$$\xi_2^2\pi^2 = 1.90(1-\xi_2^2)$$

$$\xi_2^2[\pi^2 + 1.90] = 1.90$$

$$\xi_2^2 = 0.1617$$

$$\xi_2 = 0.402$$

From eq. (3)

$$2\xi\omega_n = \frac{1}{T} \quad (T \text{ remains constant})$$

$$2\xi_1\omega_{n_1} = \frac{1}{T}$$

$$2[0.090]\omega_{n1} = \frac{1}{T}$$

$$\omega_{n1} = \frac{5.55}{T}$$

and $2\xi_2 \omega_{n2} = \frac{1}{T}$

$$2[0.402]\omega_{n2} = \frac{1}{T}$$

$$\omega_{n2} = \frac{1.25}{T}$$

From eq. (2)

$$\omega_n = \sqrt{\frac{K}{T}}$$

$$\omega_{n1} = \sqrt{\frac{K_1}{T}}$$

$$\frac{5.55}{T} = \sqrt{\frac{K_1}{T}}$$

Square on both sides in eq. (5), we get

$$\frac{30.80}{T^2} = \frac{K_1}{T}$$

$$K_1 = \frac{30.80}{T}$$

Also

$$\omega_{n2} = \sqrt{\frac{K_2}{T}}$$

$$\frac{1.25}{T} = \sqrt{\frac{K_2}{T}}$$

Square on both sides in eq. (6), we get

$$\frac{1.56}{T^2} = \frac{K_2}{T}$$

$$K_2 = \frac{1.56}{T}$$

Now reduction factor

$$RF = \frac{K_1}{K_2} = \frac{30.80/T}{1.56/T}$$

$$RF = 19.74$$

5. Attempt any one part of the following:

a. What do you mean by the root locus? State and explain all the steps used for drawing the root locus plot. (10 × 1 = 10)

Ans. i. Root locus : Refer Q. 3.10, Page 3-11C, Unit-3.
ii. Steps used for drawing root locus plot : Refer Q. 3.11, Page 3-12C, Unit-3.

b. Sketch the root locus plot and comment on stability for the system when open loop transfer function is given by :

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)(s+8)}$$

Ans. Given, $G(s)H(s) = \frac{K}{s(s+2)(s+4)(s+8)}$

- The open loop poles are at $s = 0, s = -2, s = -4, s = -8$
- There is no open loop zeros.
- Number of poles, $P = 4$
Number of zeros, $Z = 0$
 $P - Z = 4 - 0 = 4$ i.e., four branches of root locus end at infinity
- Angle of asymptotes :

$$\theta_q = \frac{(2q+1)180^\circ}{P-Z}$$

[where $q = 0, 1, 2, 3, \dots (P-Z-1)$]

$$\theta_0 = \frac{(2 \times 0 + 1) \times 180}{4 - 0} = 45^\circ$$

$$\theta_1 = \frac{(2 \times 1 + 1) \times 180}{4 - 0} = 135^\circ$$

$$\theta_2 = \frac{(2 \times 2 + 1) \times 180}{4 - 0} = 225^\circ$$

$$\theta_3 = \frac{(2 \times 3 + 1) \times 180}{4 - 0} = 315^\circ$$

5. Centroid of asymptotes :

$$\sigma = \frac{\sum \text{Real parts of poles} - \sum \text{Real parts of zeros}}{P - Z}$$

$$= \frac{(0 - 2 - 4 - 8) - (0)}{4 - 0} = \frac{-14}{4} = -3.5$$

6. Breakaway points :

The characteristic equation is, $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+2)(s+4)(s+8)} = 0$$

$$s(s+2)(s+4)(s+8) + K = 0$$

$$K = -[(s^2 + 2s)(s+4)(s+8)]$$

$$= -[(s^3 + 2s^2 + 4s^2 + 8s)(s+8)]$$

$$= -[(s^3 + 6s^2 + 8s)(s+8)]$$

$$= -[s^4 + 6s^3 + 8s^2 + 8s^3 + 48s^2 + 64s]$$

$$= -[s^4 + 14s^3 + 56s^2 + 64s]$$

$$= -s^4 - 14s^3 - 56s^2 - 64s$$

$$\frac{dK}{ds} = -[4s^3 + 42s^2 + 112s + 64] = 0$$

$$4s^3 + 42s^2 + 112s + 64 = 0$$

$$(s + 3.04)(4s^2 + 29.84s + 21.29) = 0$$

$$s = -3.04, -0.8, -6.66$$

Valid breakaway points, $s = -0.8, s = -6.66$

7. Intersection points with imaginary axis :

Characteristic equation, $1 + G(s)H(s) = 0$

$$s^4 + 14s^3 + 56s^2 + 64s + K = 0$$

Routh array :

s^4	1	56	K
s^3	14	64	0
s^2	51.42	K	
s^1	$\frac{3290.88 - 14K}{51.42}$	0	
s^0	K		

The value of K at imaginary axis,

$$\frac{3290.88 - 14K}{51.42} = 0$$

$$K_{\text{mar}} = 235.06$$

Auxiliary equation,

$$51.42s^2 + K = 0$$

$$51.42s^2 + 235.06 = 0$$

$$s^2 = \frac{-235.06}{51.42} = -4.57$$

$$s = \pm j2.13$$

8. For $0 < K < 235.06$, system is absolutely stable. At $K = 0$, system is marginally stable and for $K > 235.06$ system is unstable

Root locus plot :

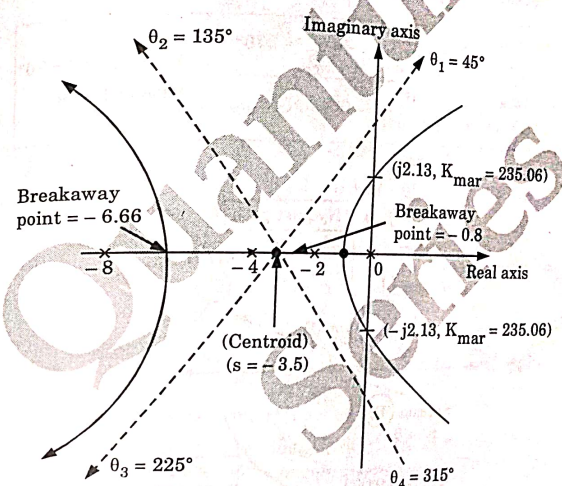


Fig. 10.

6. Attempt any one part of the following: (10 × 1 = 10)
- a. Draw the Nyquist plot and determine the stability of the system whose open loop transfer function is given as

$$G(s)H(s) = \frac{(4s+1)}{s^2(s+1)(2s+1)}$$

Ans: Given, $G(s)H(s) = \frac{4s+1}{s^2(1+s)(1+2s)}$

Step 1: No open loop pole in right of s-plane hence $P = 0$.

Step 2: $N = -P = 0$ for stability. The Nyquist plot should not encircle the critical point.

Step 3: Nyquist path is as shown in Fig. 11.

Step 4: Analysis of sections

$$G(j\omega)H(j\omega) = \frac{(4j\omega+1)}{(j\omega)^2(1+j\omega)(1+2j\omega)}$$

Section I: $s = j\omega$ to $s = +j0$

Starting point $\omega \rightarrow +\infty$	$\frac{1}{\infty} \angle \frac{90^\circ}{180^\circ 90^\circ 90^\circ} = 0 \angle -270^\circ$	$-180^\circ - (-270^\circ) = 90^\circ$
Terminating point $\omega \rightarrow +0$	$\frac{1}{0} \angle \frac{0^\circ}{180^\circ 0^\circ 0^\circ} = 0 \angle -180^\circ$	Anticlockwise

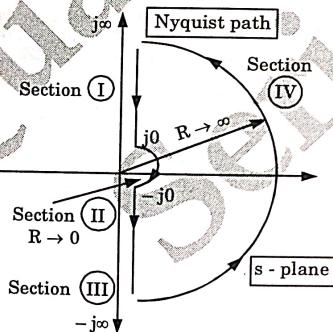


Fig. 11.

Section II: $s = +j0$ to $s = -j0$

Starting point $\omega \rightarrow +\infty$	$\infty \angle -180^\circ$
Terminating point $\omega \rightarrow -0$	$180^\circ - (-180^\circ) = 360^\circ$ Anticlockwise

Section III: Mirror image of section I about real axis.

Section IV: Not required.

Step 5: Intersection with negative real axis.

$$G(j\omega)H(j\omega) = \frac{(4j\omega+1)(1-j\omega)(1-2j\omega)}{-\omega^2(1+j\omega)(1-j\omega)(1+2j\omega)(1-2j\omega)}$$

$$= \frac{(4j\omega+1)[1-3j\omega-2\omega^2]}{-\omega^2(1+\omega^2)(1+4\omega^2)} = \frac{(1+10\omega^2)+j\omega(1-8\omega^2)}{-\omega^2(1+\omega^2)(1+4\omega^2)}$$

Equating imaginary part to zero, $1-8\omega^2 = 0$ i.e., $\omega^2 = \frac{1}{8}$

$$\therefore \omega_{pc} = \frac{1}{\sqrt{8}} = 0.3535 \text{ rad/sec}$$

Substituting in the real part, intersection point Q is,

$$Q = \frac{\left(1 + \frac{10}{8}\right)}{-\left(\frac{1}{8}\right)\left(1 + \frac{1}{8}\right)\left(1 + \frac{4}{8}\right)} = -10.667$$

Step 6: Nyquist plot is shown in the Fig. 12.

While mapping section I, the plot crosses the negative real axis at $Q = -10.667$ and then approaches to $\infty \angle -180^\circ$

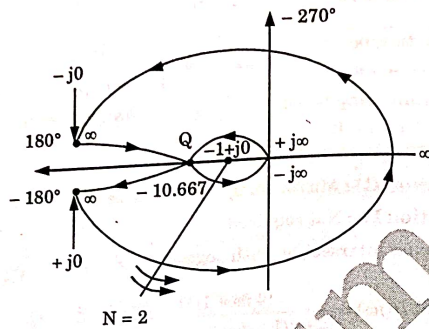


Fig. 12.

Step 7: The critical point is encircled twice in anticlockwise direction hence $N = 2$. So there are two roots in right half of s -plane. Hence the system is unstable in nature.

b. Draw the Bode plot for the open loop TF

$$G(s) = \frac{2(s+0.25)}{s^2(s+1)(s+0.5)} \text{ and from the graph determine:}$$

- Gain cross-over frequency
- Phase cross-over frequency
- Gain margin
- Phase margin
- Comment on the stability of the system.

Ans. Step 1: $G(s)H(s)$ in time constant form.

$$G(s)H(s) = \frac{2 \times 0.25 \left(1 + \frac{s}{0.25}\right)}{s^2(1+s) \times 0.5 \times \left(1 + \frac{s}{0.5}\right)} = \frac{(1+4s)}{s^2(1+s)(1+2s)}$$

Step 2: Factors

- $K = 1$, $20 \log K = 0$ dB, no effect on magnitude plot.
- $\frac{1}{s^2}$, 2 poles at origin, straight line of slope -40 dB/dec.
Passing through intersection point of $\omega = 1$ and 0 dB.
- $(1+4s)$, simple zero, $T_1 = 4$, $\omega_{C1} = \frac{1}{T_1} = 0.25$.

Straight line of slope 20 dB/dec for $\omega \geq 0.25$.

4. $\frac{1}{1+2s}$, simple pole, $T_2 = 2$, $\omega_{C2} = \frac{1}{T_2} = 0.5$.

Straight line of slope -20 dB/dec for $\omega \geq 0.5$.

5. $\frac{1}{1+s}$, simple pole, $T_3 = 1$, $\omega_{C3} = \frac{1}{T_3} = 1$.

Straight line of slope -20 dB/dec for $\omega \geq 1$.

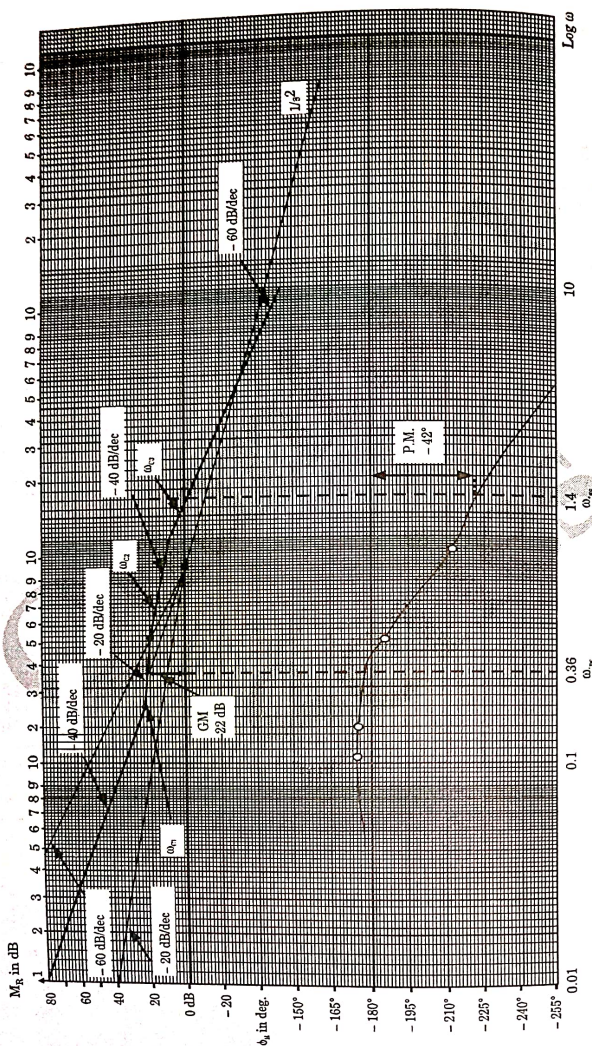
Range of ω	Resultant slope
$0 < \omega < \omega_{C1} (0.25)$	-40 dB/dec
$0.25 < \omega < \omega_{C2} (0.5)$	$-40 + 20 = -20$ dB/dec
$0.5 < \omega < \omega_{C3} (1)$	$-20 - 20 = -40$ dB/dec
$1 < \omega < \infty$	$-40 - 20 = -60$ dB/dec

Step 3: Phase angle table:

$$G(j\omega)H(j\omega) = \frac{(1+j\omega 4)}{(j\omega)^2(1+j\omega)(1+j2\omega)}$$

ω	$\frac{1}{(j\omega)^2}$	$\tan^{-1} 4\omega$	$-\tan^{-1} \omega$	$-\tan^{-1} 2\omega$	ϕ_R
0.1	-180°	21.8°	-5.71°	-11.3°	-175.22°
0.2	-180°	38.6°	-11.3°	-21.8°	-174.5°
0.5	-180°	63.4°	-26.56°	-45°	-188.16°
1	-180°	75.96°	-45°	-63.43°	-212.47°
5	-180°	87.13°	-78.69°	-84.28°	-255.84°
∞	-180°	90°	-90°	-90°	-270°

Step 4 : The Bode plot :



- Gain cross-over frequency, $\omega_{gc} = 1.4 \text{ rad/sec}$
- Phase cross-over frequency, $\omega_{pc} = 0.36 \text{ rad/sec}$
- Gain margin = -22 dB
- Phase margin = -42°
- Stability :** The gain margin and phase margin both are negative therefore, the system is unstable.

7. Attempt any one part of the following : (10 × 1 = 10)
- a. What are the different types of compensators used in control systems ? Also show that the frequency corresponding to the maximum lead angle (ω_m) is the geometric mean of the two corner frequencies of the lead compensator.

Ans.

A. Types of compensators : Refer Q. 7(a), Page SP-28C, Solved Paper (2022-23).

B. Proof :

1. Since,

$$\frac{V_o(s)}{I_i(s)} = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = \frac{\alpha(sT + 1)}{sT + 1}$$

2. Substituting, $s = j\omega$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{\alpha(j\omega T + 1)}{j\omega T + 1}$$

3. Thus the magnitude of the transfer function will be,

$$\left| \frac{V_o(j\omega)}{V_i(j\omega)} \right| = \frac{\sqrt{1 + \omega^2 T^2}}{\sqrt{1 + \alpha^2 \omega^2 T^2}}$$

4. While the phase angle will be given as,

$$\phi = \tan^{-1} \omega T - \tan^{-1} \omega \alpha T$$

5. Now determining the frequency at which ϕ is maximum

$$\frac{d\phi}{d\omega} = 0$$

$$\frac{d(\tan^{-1} \omega T - \tan^{-1} \omega \alpha T)}{d\omega} = 0$$

$$\text{Thus } \frac{\frac{1}{T}}{\omega^2 + \left(\frac{1}{T}\right)^2} - \frac{\frac{1}{\alpha T}}{\omega^2 + \left(\frac{1}{\alpha T}\right)^2} = 0$$

$$\begin{aligned} \frac{T}{1 + \omega^2 T^2} - \frac{\alpha T}{1 + \omega^2 \alpha^2 T^2} &= 0 \\ T(1 + \omega^2 \alpha^2 T^2) - \alpha T(1 + \omega^2 T^2) &= 0 \\ 1 + \omega^2 \alpha^2 T^2 - \alpha - \alpha \omega^2 T^2 &= 0 \\ \omega^2 \alpha T^2 (\alpha - 1) &= -(1 - \alpha) \\ \omega^2 \alpha T^2 &= 1 \end{aligned}$$

$$\text{Thus } \omega_m = \frac{1}{T\sqrt{\alpha}}$$

$$\omega_m = \sqrt{\frac{1}{T} \frac{1}{\alpha T}}$$

6. Therefore, at this particular frequency, the phase lead by the lead compensator will be maximum. Hence we can say the geometric mean of two corner frequencies ω_{c1} and ω_{c2} give ω_m .

$$\therefore \omega_{c1} = \frac{1}{T}, \omega_{c2} = \frac{1}{\alpha T}$$

- b. Examine the controllability and observability of a system with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = [0 \ 5 \ 1]$$

Ans. Controllability test :

$$1. \text{ We have, } A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$A^2 B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

2. Controllability test matrix is given by, $U = [B : AB : A^2 B]$

$$U = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$|U| = 0 \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 0 - 0 + [0 - 1]$$

$$|U| = -1$$

$$|U| \neq 0$$

3. Therefore, the rank of U is equal to its order i.e., 3 ($n=3$) indicating that the system is controllable.

Observability test :

1. We have,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix}, A^T = \begin{bmatrix} 0 & 0 & -4 \\ 1 & 0 & -3 \\ 0 & 1 & -2 \end{bmatrix}, C^T = \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 0 & -4 \\ 1 & 0 & -3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \\ 3 \end{bmatrix}$$

$$(A^T)^2 C^T = \begin{bmatrix} 0 & 0 & -4 \\ 1 & 0 & -3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} -4 \\ -3 \\ 3 \end{bmatrix} = \begin{bmatrix} -12 \\ -13 \\ -9 \end{bmatrix}$$

2. Observability test matrix is given by,

$$V = [C^T : A^T C^T : (A^T)^2 C^T]$$

$$= \begin{bmatrix} 0 & -4 & -12 \\ 5 & -3 & -13 \\ 1 & 3 & -9 \end{bmatrix}$$

$$V = 0 \begin{vmatrix} -3 & -13 \\ 3 & -9 \end{vmatrix} + 4 \begin{vmatrix} 5 & -13 \\ 1 & -9 \end{vmatrix} - 12 \begin{vmatrix} 5 & -3 \\ 1 & 3 \end{vmatrix}$$

$$= 0 + 4 [-45 + 13] - 12 [15 + 3]$$

$$= -128 - 216 = -344$$

3. Observability matrix $V \neq 0$, hence its rank i.e., 3 equal order i.e., 3, therefore, system is completely observable.



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